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Network coding-based reliable multicast in wireless networks

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ABSTRACT

Reliable multicast, the lossless dissemination of data from one sender to a group of receivers, has a wide range of important applications. Recently, network coding has been applied to the reliable multicast in wireless networks, where multiple lost packets with distinct intended receivers are XOR-ed together as one packet and forwarded via single retransmission, resulting in a significant reduction of bandwidth consumption. However, the simple XOR operation cannot fully exploit the potential coding opportunities and finding the optimal set of lost packets for XOR-ing is a complex NP-complete optimization problem. In this work, we intend to move beyond the simple XOR to more general coding operations. Specifically, we propose two new schemes (a static scheme which repeatedly retransmits one coding packet until all intended receivers receive it and a dynamic scheme which updates the coding packet once one or more receivers receive it) to encode packets with more general coding operations, which not only can encode lost packets with common intended receivers together to fully exploit the potential coding opportunities but also have polynomial-time complexity. We demonstrate, through both analytical and simulation results, that the proposed schemes can more greatly reduce the bandwidth requirement than the available coding-based schemes, especially in the case of high packet loss probabilities and a larger number of receivers. This reduction can vary from a few percents to over 15% depending on the packet loss probabilities and the number of receivers.

1. Introduction

Bandwidth has been one of the most precious resources in wireless networks. The *network coding* technique [1], which allows network nodes to perform coding operation in addition to the traditional routing function, has been proved promising for significantly reducing the bandwidth and energy consumptions in wireless networks.

By now, considerable efforts have been devoted to demonstrate the benefits of using network coding for different communication paradigms, such as unicast [6,2–5,7–12,14], multicast [15–17] and broadcast [18–20]. For the unicast scenario, Wu et al. [2] showed that the exchange of independent information between two nodes in a wireless network can be efficiently performed by exploiting both network coding and physical-layer broadcast. Li et al. [3,4] studied the cases of multiple unicast sessions, where network coding can only provide marginal benefits. Recently, Katti et al. [5] proposed a practical network coding-based packet forwarding architecture (called COPE) to essentially improve the network throughput of multihop wireless networks. Based on the COPE-type XOR coding scheme, coding-aware routing was proposed in Sengupta et al. [9] and [12]. Some efforts (e.g., [10,9,11]) have also been made to theoretically evaluate the throughput of COPE-type wireless networks, and Rouayheb et al. studied more general and complex coding operations rather than XOR under the name of "index coding" [13]. More recently, the physical-layer network coding was proposed to utilize wireless interference for network coding [14,7]. As for the

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multicast case, Wu et al. [15] showed that in a mobile ad hoc network, adopting network coding for minimum-cost multicast can be formulated as a linear optimization problem and solved in polynomial time. The corresponding decentralized algorithms were further proposed in Lun et al. [16] to establish the minimum-cost multicast tree. The theoretical throughput analysis of multicast with network coding has also been conducted in Park et al. [17] for unreliable ad hoc networks. Concerning the application of network coding for broadcast in wireless ad hoc networks, distributed probabilistic broadcast algorithms and deterministic broadcast algorithms have been proposed by Fragouli et al. [18,19] and Li et al. [20], respectively, resulting in a significant energy saving.

Reliable multicast [22,21], the lossless delivery of bulk data from one sender to a group of receivers, is widely used in many important applications such as the file distribution to a number of receivers and the dissemination of market data from a financial institution to its subscribers. Reliable multicast generally does not allow data loss, but can tolerate delay due to retransmissions. Traditionally, to ensure the reliable link-layer multicast the source simply retransmits one by one the *lost packets* (i.e., the packets that are not received yet by one or more receivers). Recently, Nguyen et al. [23] applied network coding to reliable link-layer multicast in wireless networks and proposed two network coding-based schemes (a static one and a dynamic one) for it. The main idea of these coding-based reliable multicast schemes is to first buffer the lost packets in the *lost-packet buffer* for some time, then, instead of transmitting these lost packets one by one, the source XORs an optimal set of lost packets with distinct intended receivers together into one packet and transmits this XOR-ed packet in one retransmission.[1] For example, suppose that a source node needs to send packets P_1, P_2 and P_3 to R_1 and R_2. The source node will first locally broadcast packets P_1, P_2 and P_3 one by one to receivers R_1 and R_2. We further suppose that R_1 successfully received P_1 and P_3, and R_2 successfully received P_1 and P_2. Since the lost packet P_2's intended receiver is R_1 and lost packet P_3's intended receiver is R_2, they have different intended receivers. Then source node will retransmit $P_2 \oplus P_3$ rather than retransmitting P_2 and P_3 separately. Upon receiving $P_2 \oplus P_3, R_1$ will XOR this coding packet with its possessed packet P_3 and consequently recover P_2. Similarly, R_2 can recover P_3 by XOR-ing the received coding packet with some of its possessed packets. The main difference between the static and dynamic schemes in Ref. [23] is that the static one will repeatedly retransmit the same XOR-ed packet until all its intended receivers successfully receive it, while the dynamic one can dynamically update the XOR-ed packet in each retransmission for a further improvement in transmission efficiency.

The adopted simple XOR operation has the advantage of encoding and decoding the packets fast, which is suitable for implementation in the networks whose node processing capability is very limited, like sensor networks. However, encoding packets with the XOR operation (over finite field $\mathbb{F}_2$) has two main limitations. First, only the lost packets with distinct intended receivers can be encoded together and thus the potential coding opportunities cannot be fully exploited. Actually, the lost packets with common intended receivers also have the potential to be encoded together by more general coding operations for transmission efficiency improvement. Second, the search for the optimal set of lost packets to XOR is very complex (actually, NP-complete), which significantly limits the scalability of these schemes.

In this paper, we intend to move beyond the simple XOR to more general coding operations, aiming to achieve a larger coding gain in ordinary wireless networks (like cellular networks). For reliable link-layer multicast in wireless networks, we propose two new coding-based schemes to conduct packet coding with more general coding operations rather than XOR, such that the above limitations of the available coding-based schemes can be significantly alleviated. In summary, the main contributions of this work are as follows:

(1) We first examine the limitations of simple XOR coding operation and then extend it to the more general coding operations. To support this extension, we propose two new schemes (also a static one and a dynamic one) to encode packets with the more general coding operations, such that the potential coding opportunities can be fully exploited while a significantly lower time complexity (polynomial time) is achieved.
(2) We provide analytical analysis to evaluate the performance in terms of both transmission efficiency and packet delay for two proposed reliable multicast schemes.
(3) We demonstrate that although the two available coding-based schemes result in a favorable reduction in the bandwidth requirement, the reduction can be more significant when the proposed schemes are applied, especially in the case of high packet loss probabilities and a large number of receivers.

The rest of this paper is organized as follows. Section 2 first introduces the limitations of the XOR coding operation and then presents two new coding-based multicast schemes. In Section 3, we analytically evaluate the transmission bandwidth and delay performance for the two proposed schemes. Numerical results obtained from the analytical model and simulation are presented in Section 4. Finally, Section 5 concludes this paper.

2. Network coding-based multicast schemes

In this section, we first introduce the limitations of the simple XOR coding operation and then present new schemes for reliable link-layer multicast in wireless networks. The new schemes encode packets with the more general coding operations rather than XOR, which not only can encode lost packets with common intended receivers together to fully exploit the potential coding opportunities but also have polynomial-time complexity.

[1] The intended receivers of a packet are the receivers that have not received this packet.

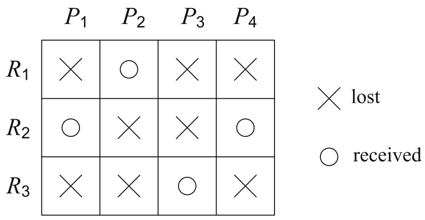

Fig. 1. An example of no coding chance when using the available coding-based schemes.

2.1. Limitations of the XOR coding operation

Despite the lower bandwidth requirements compared with the traditional non-coding scheme, both available coding-based schemes working over the XOR operation actually suffer from the following two limitations. First, only the lost packets with distinct intended receivers can be encoded together and thus the potential coding opportunities cannot be fully exploited. Actually, the lost packets with the same intended receivers also have the potential to be encoded together for transmission efficiency improvement. For example, for the pattern of lost packets in Fig. 1, there does not exist any coding chance when using an available coding-based scheme, because any two lost packets have a common intended receiver. Whether the static one or the dynamic one is used, the source needs to retransmit at least three times in this example. However, by encoding packets with more general operations (to be discussed in Section 2.2), these lost packets can actually be transmitted within fewer retransmissions.

Second, finding the maximum set of lost packets with distinct intended receivers to XOR is actually a very complex problem, which will significantly limit its scalability. Let L be the number of lost packets. Without loss of generality, assume that $P_1, P_2, \ldots, P_L$ are lost packets. Then, this optimization problem can be mathematically formulated as follows.

Given: values of $e_{i,j}$'s: $\quad i \in \{1, \ldots, M\}, j \in \{1, \ldots, L\}$.
Encoded packet: $P = a_1P_1 \oplus \cdots \oplus a_LP_L$
Maximize: $\sum_{i=1}^{L} a_i$
Over variables: $a_i \in \{0, 1\} : 1 \leqslant i \leqslant L$
Subject to: $\sum_{i=1}^{L} a_i e_{1,i} \leqslant 1,$
$\sum_{i=1}^{L} a_i e_{2,i} \leqslant 1,$
$\ldots$
$\sum_{i=1}^{L} a_i e_{M,i} \leqslant 1.$

Below, we show that this maximum lost-packet coding (MLPC) problem is NP-complete based on the reduction from the NP-complete maximum independent set (MIS) problem [24].

Theorem 1. *The MLPC problem is NP-complete.*

Proof. It is easy to conclude that the MLPC problem belongs to NP. Therefore, it is enough to show a polynomial-time reduction from the MIS problem described below to the MLPC problem.

Maximum Independent Set Problem:

Instance: A graph $G(V, E)$ and a positive integer $K \leqslant |V|$.
Question: Does G contain a subset of vertices with cardinality K such that no two vertices in this subset are adjacent in G?

Here is the reduction. Given an instance $G = (V, E)$ of the MIS problem, construct an instance of the MLPC problem as follows. Label the nodes in G by $v_1, v_2, \ldots, v_{|V|}$. Then the lost packet set is defined as $\{P_1, P_2, \ldots, P_{|V|}\}$, where P_i corresponds to the vertex v_i in the MIS problem. Let $e_{i,j} = 1$ mean that R_i did not correctly receive P_j and $e_{i,j} = 0$ mean that R_i correctly received P_j. At the beginning, set each $e_{i,j}$ to zero and set parameter k to zero. Now, in the order from $i = 1$ to $i = |V|$, we define the receivers that do not correctly receive P_i in the following way: corresponding to each v_i's neighbor v_j with $j > i$, let $k = k + 1, e_{k,i} = 1$ and $e_{k,j} = 1$.

Based on the above construction, we can know that the answer to the instance of the MIS problem is "YES" iff there is a set of K lost packets from different receivers in the MLPC problem. □

The main notations employed in the proposed schemes and the performance analysis in Section 3 are summarized in Table 1.

2.2. Static general-coding-based (SGC) scheme

Similar to the available coding-based schemes, this scheme also consists of the *transmission phase* and

Table 1
Main notations employed in this paper.

Notation	Meaning
Common notations	
R_0	Source node
R_i	Receiver i ($i \geqslant 1$)
M	Number of receivers
p_i	Packet delivery ratio of wireless link (R_0, R_i)
N	Number of packets of each generation
N_l	Number of lost packets in a generation
N_r	Total number of retransmissions for a generation
$e_{i,j}$	Indicator about whether R_i correctly receives P_j or not. It equals zero if R_i correctly receives P_j; otherwise, it equals one
$\mathbf{b}(P_C, A)$	Coding vector of the encoded packet P_C over packet set A
Special notations for static scheme	
S_p	A set p of lost packets to be encoded together for retransmission
$N_l^{p,i}$	Number of lost packets in the set S_p which are not received at R_i
$N_r^{p,i}$	Number of retransmissions until R_i receive exactly $N_l^{p,i}$ packets, during the retransmission of lost packets in S_p
N_r^p	Total number of retransmissions for a set S_p of lost packets
Special notations for dynamic scheme	
S	Set of lost packets in a generation and $N_l = \|S\|$
S_d	Set of lost packets to be encoded for the current retransmission
V_i	Set of coding vectors for the packets that have already been received by R_i
N_t	Total number of transmissions (including retransmissions) for a generation

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retransmission phase. The transmission phase of this scheme is the same as the old one, in which the source just simply transmits a fixed number of packets one by one. All these packets are called a *generation* in this paper.

During the retransmission phase, rather than using a complex (NP-complete) algorithm to find the optimal set of lost packets for XOR-ing as the old scheme does, here we first adopt a simple approach (Procedure 1 below) to group all lost packets into different sets. Then, the source will cope with the lost packets set by set. Only when all intended receivers have recovered all lost packets of the current set, the source will continue to the next set. During the retransmission of each set of lost packets (denoted by S_p), after a necessary parameter initialization (Procedure 2), we use a novel approach (Procedure 3) to determine the proper combination of these lost packets for an efficient lost-packet retransmission. After the resulting coding packet is transmitted, if some receivers still cannot recover all packets of S_p, the SGC scheme will update some related parameters (by Procedure 4) and then continue to obtain an new coding packet (by Procedure 3) to transmit. In the following, we introduce in detail the main procedures of the retransmission phase in the SGC scheme.

At the beginning of retransmission phase, the source first conducts the following operation.

Procedure 1 (Lost packets grouping): Suppose N_l packets are lost in the current generation. We group these N_l lost packets into $\lfloor \frac{N_l}{M} \rfloor + 1$ sets,[2] such that $\lfloor \frac{N_l}{M} \rfloor$ sets have the same cardinality M and the last set has cardinality (N_l mod M). For the last set with cardinality (N_l mod M), add additional $M - (N_l \bmod M)$ packets with only bits zero into this set, and also set all indicators $e_{i,j}$ of these additional packets to zero.

Notice that, unlike the available static schemes where only lost packets with distinct intended receivers will be XOR-ed together, here all lost packets in the same set are encoded together over a large enough finite field for retransmission, no matter whether these packets have common intended receivers or not. The determination of the finite field size will be introduced later. Due to the adoption of general coding operation (over a selected finite field) rather the simple XOR, for the same set of lost packets, the source is able to transmit different coding packets for different retransmissions which is unachievable when using the simple XOR. In this way, the potential coding opportunities can be exploited more efficiently. Let us still consider the example in Fig. 1. Using the SGC scheme, the source will group the lost packets P_1, P_2 and P_3 into a set. Suppose the selected finite field is $\mathbb{F}_{2^2} = \{0, 1, 2, 3\}$, where each element represents a stream of two bits (e.g., element 3 represents 11). Then the source can first transmit the coded packet $P_1 + P_2 + P_3$, and then $P_1 + 2P_2 + 3P_3$. Once R_1 receives these two coded packets, it can decode P_1 and P_3 by Gaussian elimination. Similarly, R_2 and R_3 can recover all packets. In this way, it is possible to finish the retransmission of P_1, P_2 and P_3 within only two times rather than at least three times as in the old non-coding scheme.

[2] Without loss of generality, we suppose that (N_l mod M) is not equal to zero.

For a set of native packets $A = \{P_1, \ldots, P_k\}$ (i.e., the packets without encoding) and one of its encoded packet $P_C = \sum_{i=1}^{k} g_i P_i$ over a finite field $\mathbb{F}_q$ with the base q (i.e., $g_i \in \mathbb{F}_q$), we call $(g_1, \ldots, g_k)P_C$'s *coding vector* over A, and denote it by $\mathbf{b}(P_C, A)$. Thus, the main problem now is the selection of coding vector $(g_1, \ldots, g_k)$ for each retransmission. Before retransmitting each set of lost packets, the source needs to first conduct the following parameter initialization.

Procedure 2 (Parameter initialization): For a given set S_p of lost packets, let $N_l^{p,i}$ be the number of packets in S_p that R_i has not received yet. Initialize the value of $N_l^{p,i}$ by $N_l^{p,i} = \sum_{P_j \in S_p} e_{i,j}, \forall i \in \{1, \ldots, M\}$. Also, initialize the set V of coding vectors as $V = V_{M,q} \setminus \{(0^{i-1}, 1, 0^{M-i}) : i \setminus in\{1, \ldots, M\}$ and and ith lost packet in S_p has been received by at least one receiver$\}$, where $V_{M,q}$ is the maximum set of M-dimensional vectors over finite field $\mathbb{F}_q$, which contains M distinct unit vectors $(1,0,\ldots,0), \ldots (0,0,\ldots,1)$ and any M vectors of it are linearly independent. The construction of $V_{M,q}$ has been widely studied in the field of the systematic maximum-distance separable (MDS) codes [25,26].

After the above parameter initialization, now the source can select the coding vector for each retransmission.

Procedure 3 (Coding vector selection): Randomly select a vector $\mathbf{v}$ in V and let $V \leftarrow V \setminus \{\mathbf{v}\}$. Then let vector $\mathbf{v}$ be a coding vector over S_p to obtain an encoded packet.

A native or encoded packet received by a network node is said to be *non-innovative* (*innovative*) for this node if this packet is available or can be (not available and cannot be) generated by linear combination of its previously received packets. Thus, the receiver R_i needs to receive at least $N_l^{p,i}$ innovative packets to recover all its lost packets in S_p. During the retransmission for lost packets in S_p, a receiver R_i that has not received $N_l^{p,i}$ innovative packets is said to be *unsaturated*. Note that for each unsaturated receiver, the coding vector selected in Procedure 3 is independent of the coding vectors of its previously received packets, i.e., the resulting encoded packet is innovative to it. Clearly, this coding approach minimizes the expected number of retransmissions required for the delivery of lost packets in S_p.

After retransmitting an encoded packet, the source needs to update the parameters $N_l^{p,i}$ and V as follows according to the feedback from the receivers.

Procedure 4 (Parameter update): For each unsaturated receiver R_i (with $N_l^{p,i} \geqslant 1$), if it correctly receives $P_C, N_l^{p,i} \leftarrow N_l^{p,i} - 1$. For each encoded packet P_j of $S_p \bigcup \{$transmitted encoded packets from $S_p\}$, if $\sum_{i:N_l^{p,i} \geqslant 1} e_{i,j} = 0$, then the coding vector of P_j can be reused and thus $V \leftarrow V \bigcup \{\mathbf{b}(P_j, S_p)\}$.

Summarizing the above procedures, the SGC scheme is formally illustrated in Fig. 2.

Next, we discuss the necessary size of field $\mathbb{F}_q$ and also the complexity of this scheme.

2.2.1. Field size

As the number of receivers M increases, the necessary cardinality of the adopted $V_{M,q}$ (and thus the necessary size of $\mathbb{F}_q$) also increases. The following theorem shows the sufficient and necessary condition on the required field size.

Procedure of the SGC scheme
Steps:

```
Transmit N native packets one by one and build the packet-loss table.
Conduct Procedure 1 to group N_l lost packets into k sets.
for j = 1 to k do
    Let S_p be the j'th set of lost packets.
    Conduct Procedure 2 to initialize parameters N_l^{p,i} and V.
    while exist one or more unsaturated receivers (i.e. ∃i, N_l^{p,i} > 0) do
        Conduct Procedure 3 to select a coding vector and obtain an encoded packet P_C.
        Repeatedly transmit packet P_C until at least one unsaturated receiver receives it.
        Conduct Procedure 4 to update parameters N_l^{p,i} and V.
    end while
end for
```

Fig. 2. New static multicast scheme.

	P_1	P_2	P_3	P_{C_1}	P_{C_2}	P_{C_3}
R_1	○	○	×	×	×	×
R_2	×	×	×	○	○	×
R_3	×	×	○	×	×	○

Fig. 3. A packet loss example needing the maximum number of innovative packets.

Theorem 2. *For a given number of receivers M, the proposed static scheme can always guarantee an innovative packet for all unsaturated receivers if and only if q satisfies* $|V_{M,q}| > M(M-1)$.

Proof. The maximum $V_{M,q}$ is needed when the following worst case happens: each transmitted packet is received by exactly one receiver and each receiver has received $M-1$ packets, as illustrated in Fig. 3. In this worst case, $M(M-1)$ innovative packets have already been transmitted. If we have one more packet innovative to all receivers to transmit, then once a receiver R_i receives this innovative packet, R_i can recover all M packets and does not need to be considered any more. Then any packet previously received by R_i can be used for retransmission, which is innovative to all remaining unsaturated receivers. □

2.2.2. Computational complexity

Here, we briefly analyze the computational complexity of obtaining an encoded packet for transmission when using the SGC scheme. During the transmission phase, the source just transmits a native packet, which takes only constant time. During the retransmission phase, the source first needs time $O(M^2)$ to get S_p and calculate $N_l^{p,i}$. Then, for each retransmission, the source takes time $O(M^2)$ to linearly combine M lost packets, takes time $O(M)$ to update parameter $N_l^{p,i}$ and takes time $O(M^3)$ to update parameter V. Thus, the overall computational complexity of obtaining an encoded packet for retransmission is $O(M^3)$.

2.3. Dynamic general-coding-based (DGC) scheme

The DGC scheme also consists of the transmission phase and retransmission phase. Similar to the SGC scheme, the DGC scheme also relaxes the restricted coding principle and uses a simple algorithm to find the set of lost packets for encoding. The main difference between them is that in the DGC scheme, the encoded packet is dynamically updated for each retransmission such that the potential coding opportunities can be exploited more effectively. Let us still consider the example in Fig. 1. Same as the SGC scheme, the source can first transmit the encoded packet $P_1+P_2+P_3$ and then $P_1+2P_2+3P_3$ which are built over the finite field $\mathbb{F}_{2^2}=\{0,1,2,3\}$. Suppose that R_1 has recovered all native packets (i.e., P_1 and P_3) inside the current coding packet. Different from the SGC scheme, the DGC scheme will find a new encoded packet (say $P_2+2P_3+P_4$) which is still innovative to all receivers. It is notable, however, that due to the new requirement of the dynamic update of encoded packet, now the main grouping process and also the selection process of coding vector in the retransmission phase become very different.

Basically, before starting to encode lost packets for transmission, the DGC scheme first conducts parameter initialization by Procedure 1 below. After this step, the DGC scheme will determine the set S_p of lost packets for coding by Procedure 2, and then obtain the coding vector (i.e., the coding packet) by Procedure 3. After the transmission of coding packet, the DGC scheme continues to update the S_p by Procedure 2 according to the reception information fed back from the receivers, and then use Procedure 3 to obtain a new coding packet to transmit. This process is repeatedly conducted until all receivers recover all lost packets. In the following, we introduce the DGC scheme in detail.

At the beginning of the retransmission phase, the source first conducts the following operation.

Procedure 1 (Parameter initialization): Let S be the set of lost packets in the current generation, S_d be the set of packets to be encoded for the current retransmission, and V_i be the set of coding vectors of the encoded packets that are already received by R_i. Initialize S_d and V_i $(i=1,\ldots,M)$ as the empty set.

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Procedure of the DGC scheme
Steps:
1 Transmit N native packets one by one and build the packet-loss table.
2 Conduct Procedure 1 to initialize parameters S, S_d and V_i $(i = 1, \ldots, M)$.
3 **while** $S \neq \phi$ and $S_d \neq \phi$ **do**
4 Conduct Procedure 2 to update S_d.
5 Conduct Procedure 3 to obtain $\mathbf{y}'$, which is independent of each V_i satisfying $|V_i| < |S_d|$, and obtain the encoded packet P_C.
6 Repeatedly transmit packet P_C until one or more receivers receive it.
7 For any R_i that correctly receives P_C, $V_i \leftarrow V_i \bigcup \{\mathbf{y}'\}$.
8 **end while**

Fig. 4. New dynamic multicast scheme.

At each retransmission, we need to determine the set S_d and also the coding vector over S_d to get the encoded packet.

Procedure 2 (Determination of S_d): For each receiver R_i, check whether its $|V_i|$ is equal to $|S_d|$. If we cannot find an R_i with $|V_i|$ equal to $|S_d|$, the S_d for the current transmission remains unchanged, just same as last transmission. Otherwise, the source updates S_d for the current transmission by removing some packets from and adding some packets in it as follows.

- *Updating $e_{i,j}$:* For each receiver R_i with $|V_i|$ equaling $|S_d|$ and each $P_k \in S_d$, set $e_{i,k}$ as 0 since R_i has already recovered all lost packets in S_d.
- *Packet-removing:* For any packet $P_j \in S_d$ satisfying $\sum_{k=1}^{M} e_{k,j} = 0$, first conduct the following coding vector update: for each R_i and each vector $\mathbf{v} = (v_1, \ldots, v_{|S_d|}) \in V_i$, remove from $\mathbf{v}$ the entry corresponding to packet P_j and if the resulting $\mathbf{v} = \mathbf{0}$, let $V_i \leftarrow V_i \setminus \{\mathbf{v}\}$. Second, remove this packet from S_d.
- *Packet-adding:* For each packet P_n in S, conduct the following operations: check whether there exists at least one receiver R_i satisfying $\sum_{k:P_k \in S_d} e_{i,k} = 0$and $e_{i,n} = 1$. If so, first add packet P_n into S_d and remove P_n from S; then for each R_j and each $\mathbf{v} = (v_1, \ldots, v_{|S_d|-1}) \in V_j$, add a new entry of zero at the end of $\mathbf{v}$ and if $e_{j,n} = 0$ add the $|S_d|$-dimensional unit vector $(0, \ldots, 0, 1)$ into V_i.

With the set S_d, the determination of coding vector over S_d is done as follows.

Procedure 3 (Determination of coding vector): First, for each receiver R_i with $|V_i| < |S_d|$, obtain a vector $\mathbf{b}_i$ which is independent of V_i by using the Gaussian elimination method and generate an orthogonal set V_i' through orthogonalizing the vectors of V_i. Then, for each obtained vector $\mathbf{b}_i, \mathbf{y}_i \leftarrow \mathbf{b}_i - \sum_{\mathbf{v} \in V_i'} \frac{\langle \mathbf{b}_i, \mathbf{v} \rangle}{\|\mathbf{v}\|} \mathbf{v}$. Finally, with the obtained $\mathbf{y}_i$, we can use the approach introduced in Lemma 7 of [27] to obtain a coding vector $\mathbf{y}'$ that satisfies $\mathbf{y}' \cdot \mathbf{y} \neq 0$ for each $\mathbf{y}$.

The following lemma shows that the obtained $\mathbf{y}'$ is linearly independent of each V_i.

Lemma 1. *Let B denote a set of n-dimensional vectors. Vector $\mathbf{a}$ is orthogonal to B, i.e., $\mathbf{a} \cdot \mathbf{b} = 0$ for any $\mathbf{b} \in B$. Then if vector $\mathbf{x}$ satisfies $\mathbf{x} \cdot \mathbf{a} \neq 0, \mathbf{x}$ is linearly independent of B.*

Procedure 3 guarantees that the selected coding vector is independent of the coding vectors for the received packets of this receiver. Clearly, this dynamic coding way can minimize the average number of retransmissions per generation.

Formally, the DGC scheme is shown in Fig. 4. Next, we briefly discuss the necessary field size and the computational complexity of this scheme.

2.3.1. Field size

The following lemma (from Lemma 6 in [27]) and corollary show a sufficient condition on the necessary size of the field $\mathbb{F}_q$.

Lemma 2. *Let $\mathbb{F}^h$ be the space of h-dimensional vectors over $\mathbb{F}$. If $|\mathbb{F}| \geqslant n$ and vector pairs $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbb{F}^h \times \mathbb{F}^h$ satisfy $\mathbf{x}_i \cdot \mathbf{y}_i \neq 0$ for each $i \in \{1, \ldots, n\}$, then there is a linear combination $\mathbf{u}$ of $\mathbf{x}_1, \ldots, \mathbf{x}_n$ such that $\mathbf{u} \cdot \mathbf{y}_i \neq 0$ for each i.*

Corollary 1. *If $|\mathbb{F}| \geqslant M$ and $n \leqslant M$, then for vectors $(\mathbf{y}_1, \mathbf{y}_1), \ldots, (\mathbf{y}_n, \mathbf{y}_n) \in \mathbb{F}^h \times \mathbb{F}^h$ there is a linear combination $\mathbf{y}'$ of $\mathbf{y}_1, \ldots, \mathbf{y}_n$ such that $\mathbf{y}' \cdot \mathbf{y}_i \neq 0$ for each i.*

Similarly to the SGC scheme, this DGC scheme conducts the coding operation over a general finite field $\mathbb{F}_q$ rather than over $\mathbb{F}_2$. The following theorem shows the sufficient condition on the size of $\mathbb{F}_q$ for this scheme.

Theorem 3. *Given the value of M, if $q \geqslant M$, then in the DGC scheme we always have a packet innovative to all unsaturated receivers for retransmission.*

Proof. Based on Corollary 1, we can easily arrive at the result. □

2.3.2. Computational complexity

Here, we analyze the computational complexity of obtaining a packet for transmission when using the DGC scheme. During the transmission phase, the source just transmits a native packet, which takes only constant time. During Procedure 2 of the retransmission phase, updating $e_{i,j}$ takes time $O(MN)$, removing packets from S_d and updating related parameters take time $O(MN^3)$, and adding packets into S_d and updating related parameters take time $O(M^2N^3)$. In Procedure 3, Gaussian elimination, Gram–Schmidt orthonormalization process and the calculation

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of $\mathbf{y}'$ take time $O(MN^3), O(MN^3)$ and $O(MN^2)$, respectively. Thus, the overall computational complexity is $O(M^2N^3)$.

3. Performance analysis

In this section, we conduct the theoretical analysis of new schemes in terms of the transmission efficiency and the delay performance. By transmission efficiency, the same metric called *transmission bandwidth* as in Nguyen et al. [23] is adopted, which is defined as the average number of transmissions required to successfully transmit a packet to all receivers. By delay performance, we will evaluate the average number of transmissions a packet needs to wait from when this packet is transmitted for the first time until it is successfully received by all receivers (referred to *retransmission delay* in this paper).

Before going into the detailed performance analysis of proposed schemes, we first give one lower bound on the transmission bandwidth such that we can have some idea about how efficient our proposed schemes are (which will be discussed in Section 4).

Theorem 4. *For the reliable multicast with M receivers, denote the packet delivery rate at receiver R_i by p_i. Then the transmission bandwidth η is lower bounded as follows:*

$$\eta \geqslant \frac{1}{\min\limits_{1\leqslant i\leqslant M} p_i}. \tag{1}$$

Proof. This is easy to derive. Let us consider the receiver with the minimal packet delivery rate among all receivers. We can easy to know that, for this receiver, the average number of transmission necessary for successfully receiving one packet is $\frac{1}{\min_{1\leqslant i\leqslant M} p_i}$.

Since there are other receivers which also need to receive the packets from the source, the average number of transmission necessary for successfully receiving one packet at all receivers is clearly at least $\frac{1}{\min_{1\leqslant i\leqslant M} p_i}$. □

3.1. Analysis of the SGC scheme

We first provide the analysis for the SGC scheme.

3.1.1. Transmission bandwidth

Denote by η_g the transmission bandwidth when using the proposed SGC scheme and by N_r the number of retransmission packets for a generation of lost packets. Then η_g is given by

$$\eta_g = E[(N+N_r)/N] = 1 + \frac{1}{N}E[N_r] = 1 + \frac{1}{N}\sum_{L=0}^{N} P[N_l = L]E[N_r|N_l = L], \tag{2}$$

where N_l the total number of lost packets among a generation of packets.

In the above equation, under the assumption that the packet loss probabilities of different links are independent from one another, the $P[N_l = L]$ can be easily evaluated by

$$P[N_l = L] = \binom{N}{L}\left(1 - \prod_{n=1}^{M} p_n\right)^L \left(\prod_{n=1}^{M} p_n\right)^{N-L}, \tag{3}$$

where p_n is the packet delivery ratio of wireless link (R_0, R_n) and $\prod_{n=1}^{M} p_n$ is the probability that a packet is successfully received by all receivers.

We now analyze the conditional expected number of retransmissions $E[N_r|N_l = L]$ in Eq. (2). In the static scheme, L lost packets are grouped into $\lfloor \frac{L}{M} \rfloor + 1$ sets of lost packets, $\lfloor \frac{L}{M} \rfloor$ sets with cardinality M and one set with cardinality $L \bmod M$. Since the sets with the same cardinality M have the same expected number of retransmissions, so $E[N_r|N_l = L]$ is given by:

$$E[N_r|N_l = L] = \left\lfloor \frac{L}{M} \right\rfloor E[N_r^p||S_p| = M] + E[N_r^p||S_p| = L \bmod M], \tag{4}$$

where S_p denotes the set of lost packets that are encoded together for retransmission and N_r^p is the number of retransmission packets for S_p.

So far, the work left for evaluating η_g is the calculation of $E[N_r^p||S_p| = k]$. It is given by the following formula:

$$E[N_r^p||S_p| = k] = \sum_{i=1}^{\infty} i \cdot P(N_r^p = i||S_p| = k) = \sum_{i=1}^{\infty} i \cdot \left(P(N_r^p \leqslant i||S_p| = k) - P\left(N_r^p \leqslant i-1\,\middle|\,|S_p| = k\right)\right). \tag{5}$$

Denote by $N_l^{p,i}$ the number of unreceived packets at R_i in a set S_p of lost packets. For a set of lost packets, the number of retransmission packets N_r^p is

$$N_r^p = \max_{j\in\{1,\ldots,M\}} N_r^{p,j},$$

where $N_r^{p,j}$ is a random variable denoting the number of transmissions required for R_j to receive $N_l^{p,j}$ packets. Then we have

$$\begin{aligned} P[N_r^p \leqslant i||S_p| = k] &= P\left(N_r^{p,1} \leqslant i, \ldots, N_r^{p,M} \leqslant i\,\middle|\,|S_p| = k\right) \\ &= \sum_{0\leqslant i_1,\ldots,i_M \leqslant \min\{i,k\}\, i_1+\cdots+i_M \geqslant k} \times P\left(N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M, N_r^{p,1} \leqslant i, \ldots, N_r^{p,M} \leqslant i\,\middle|\,|S_p| = k\right) \\ &= \sum_{\substack{0\leqslant i_1,\ldots,i_M \leqslant \min\{i,k\} \\ i_1+\cdots+i_M \geqslant k}} \times P\left(N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\,\middle|\,|S_p| = k\right) \cdot P\left(N_r^{p,1} \leqslant i, \ldots, N_r^{p,M} \leqslant i\,\middle|\,|S_p| = k, N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right), \quad i = 1, 2, \ldots \end{aligned} \tag{6}$$

In the second equality above, at each receiver R_j, the number $N_l^{p,j}$ of its lost packets among S_p must be smaller than or equal to the total number i of retransmissions and also the number k of lost packets of the whole set. Additionally, it is clear that the summarization of $N_l^{p,1}, \ldots, N_l^{p,M}$ must be large than or equal to k. The second term in the last equality above can be evaluated as follows:

$$\begin{aligned} &P\left(N_r^{p,1} \leqslant i, \ldots, N_r^{p,M} \leqslant i\,\middle|\,|S_p| = k, N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right) \\ &= P\left(N_r^{p,1} \leqslant i|N_l^{p,1} = i_1\right) P\left(N_r^{p,2} \leqslant i|N_l^{p,2} = i_2\right) \ldots \times P\left(N_r^{p,M} \leqslant i|N_l^{p,M} = i_M\right) \\ &= \prod_{j=1}^{M} \sum_{m=i_j}^{i} \binom{m-1}{m-i_j} p_j^{i_j} (1-p_j)^{m-i_j}. \end{aligned} \tag{7}$$

In the above, the first equality follows from the assumption that the packet loss at the receivers is independent.

About the evaluation of $P(N_l^{p,1}=i_1,\ldots,N_l^{p,M}=i_M \big| |S_p|=k)$ in Eq. (6), we have the following lemma.

Lemma 3. *For k packets, given that each of them is not correctly received by at least one receiver, the probability that R_n $(n=1,\ldots,M)$ did not correctly receive i_n packets among these k packets is given by*

$$P\left(N_l^{p,1}=i_1,\ldots,N_l^{p,M}=i_M \Big| |S_p|=k\right) = \left(\prod_{n=1}^{M} p_n^{k-i_n}(1-p_n)^{i_n}\right) \cdot \left(\sum_{l=0}^{k-\max_{n\in\{1,\ldots,M\}} i_n} (-1)^l \binom{k}{l} \prod_{n=1}^{M} \binom{k-l}{i_n}\right) \Bigg/ \left(1-\prod_{n=1}^{M} p_n\right)^k. \tag{8}$$

Proof. See Appendix A. □

Now, by substituting Eqs. (8) and (7) into (6) and substituting Eq. (6) into (5), we have

$$E[N_r^p \big| |S_p|=k] = \sum_{i=1}^{\infty} i(q(i,k)-q(i-1,k)), \tag{9}$$

where

$$q(i,k) = \sum_{0\leqslant i_1,\ldots,i_M\leqslant \min\{i,k\} i_1+\cdots+i_M\geqslant k} \times \left(\prod_{j=1}^{M}\sum_{m=i_j}^{i}\binom{m-1}{m-i_j} p_j^{i_j}(1-p_j)^{m-i_j}\right) \cdot \left(\prod_{n=1}^{M} p_n^{k-i_n}(1-p_n)^{i_n}\right) \cdot \left(\sum_{l=0}^{k-\max_{n\in\{1,\ldots,M\}} i_n} (-1)^l \binom{k}{l} \prod_{n=1}^{M} \times \binom{k-l}{i_n}\right) \Bigg/ \left(1-\prod_{n=1}^{M} p_n\right)^k. \tag{10}$$

Finally, we summarize the evaluation of η_g as the following theorem.

Theorem 5. *The transmission bandwidth η_g of the proposed static scheme with M receivers and lost-packet buffer size N is:*

$$\eta_g = 1+\frac{1}{N}\sum_{L=0}^{N}\left\{f\left(\prod_{n=1}^{M} p_n, L, N\right) \cdot \sum_{i=1}^{\infty}\left(i\cdot\left\lfloor\frac{L}{M}\right\rfloor (q(i,M)-q(i-1,M)) + i\cdot(q(i,L \bmod M) - q(i-1,L \bmod M))\right)\right\}, \tag{11}$$

where $q(i,k)$ is given by Eq. (10) *and*

$$f(p,i,j) = \binom{j}{i} p^{j-i}(1-p)^i. \tag{12}$$

Proof. Combining Eqs. (2), (3), (4) and (9), we easily get the result. □

3.1.2. Retransmission delay

Denote by γ_g the retransmission delay when using the proposed SGC scheme. It is easy to know that the larger the lost-packet buffer size N, the larger the retransmission delay γ_g.

To decode the received encoded packets, every receiver will perform Gaussian elimination after every received innovative packet to ensure the earliest possible decoding. Because different selections of innovative packets for transmission will lead to different results of Gaussian elimination (i.e., different packet delay) at receivers, so the exact analysis of the retransmission delay is quite difficult. Here we present an upper bound on the retransmission delay in the following theorem.

Theorem 6. *The retransmission delay γ_g of the proposed static scheme is upper bounded by*

$$\gamma_g < \frac{1}{N}\sum_{L=0}^{N}\left\{f\left(\prod_{n=1}^{M} p_n, L, N\right) \cdot \left(\frac{(N-1)L}{2} + 0.5\left\lfloor\frac{L}{M}\right\rfloor (M+L+L\%M) \cdot \sum_{i=1}^{\infty} i\cdot(q(i,M)-q(i-1,M)) + (L\%M)\sum_{i=1}^{\infty} i \cdot (q(i,L\%M)-q(i-1,L\%M))\right)\right\}, \tag{13}$$

where $q(i,k)$ and $f(p,i,j)$ are shown in Eqs. (10) and (12)*, respectively, and the symbol % represents the integer modulo operation.*

Proof. See Appendix B. □

3.2. Analysis of the DGC scheme

Here we evaluate the transmission bandwidth and retransmission delay of the DGC scheme.

3.2.1. Transmission bandwidth

Denote by η_d the transmission bandwidth when using the proposed DGC scheme. The transmission efficiency η_d of the proposed dynamic scheme is given in the following theorem.

Theorem 7. *The transmission bandwidth η_d of dynamic scheme with M receivers and lost-packet buffer size N is*

$$\eta_d = \frac{1}{N}\sum_{i=N}^{\infty} i\left(\prod_{j=1}^{M}\sum_{k=N}^{i} P_{j,k} - \prod_{j=1}^{M}\sum_{k=N}^{i-1} P_{j,k}\right), \tag{14}$$

where $P_{j,k} = \binom{k-1}{N-1} p_j^N (1-p_j)^{k-N}$.

Proof. Let N_i be a random variable denoting the number of transmissions for receiver R_i to successfully receive N packets. Clearly, $N_i \geqslant N$. Then the total number of transmissions to guarantee that all receivers successfully receive N packets is

$$N_t = \max_{j\in\{1,\ldots,M\}} N_j.$$

The average number of transmissions required to successfully transmit a packet to all receivers is given by

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$$\eta_d = \frac{1}{N}E[N_t] = \frac{1}{N}\sum_{i=N}^{\infty} iP[N_t = i]$$
$$= \frac{1}{N}\sum_{i=N}^{\infty} i(P[N_t \leqslant i] - P[N_t \leqslant i-1])$$
$$= \frac{1}{N}\sum_{i=N}^{\infty} i(P[N_1 \leqslant i,\ldots,N_M \leqslant i] - P[N_1 \leqslant i-1,\ldots,N_M \leqslant i-1])$$
$$= \frac{1}{N}\sum_{i=N}^{\infty} i\left(\prod_{j=1}^{M} P[N_j \leqslant i] - \prod_{j=1}^{M} P[N_j \leqslant i-1]\right). \quad (15)$$

In the above equation, $P[N_j \leqslant i]$ is given by

$$P[N_j \leqslant i] = \sum_{k=N}^{i} P[N_j = k] = \sum_{k=N}^{i} \binom{k-1}{N-1} p_j^N (1-p_j)^{k-N}. \quad (16)$$

Finally, substituting Eq. (16) into (15), we arrive at the result. □

3.2.2. Retransmission delay

Denote by γ_d the retransmission delay when using the proposed DGC scheme. The following theorem shows the delay performance of the proposed dynamic scheme.

Theorem 8. *The retransmission delay γ_d of the proposed dynamic scheme is upper bounded by*

$$\gamma_d < \frac{1}{N}\sum_{L=0}^{N}\left\{ f\left(\prod_{n=1}^{M} p_n, L, N\right) \cdot \left(\frac{(N-1)L}{2} + L\sum_{i=1}^{\infty} i\cdot(q(i,M) - q(i-1,M))\right)\right\}, \quad (17)$$

where $q(i,k)$ and $f(p,i,j)$ are shown in Eqs. (10) and (12), respectively.

Proof. Same as the proposed static scheme, the retransmission delay of the proposed dynamic scheme is given by (see the proof of Theorem 13)

$$\gamma_d = E(D) = \frac{1}{N}\left(E\left[\sum_{P_i\in S} i\right] + E\left[\sum_{P_i\in S} D_i\right]\right). \quad (18)$$

In the above equation, $E[\sum_{P_i\in S} i]$ is already given in Eq. (28), and $E[\sum_{P_i\in S} D_i]$ is upper bounded by

$$E\left[\sum_{P_i\in S} D_i\right] = \sum_{L=0}^{N}\left(P[|S|=L]E\left[\sum_{P_i\in S} D_i||S|=L\right]\right) \leqslant \sum_{L=0}^{N}\left[\binom{N}{L} \times \left(1-\prod_{n=1}^{M} p_n\right)^{L}\left(\prod_{n=1}^{M} p_n\right)^{N-L}(L\cdot E[N_r^p||S_p|=L])\right]. \quad (19)$$

Combining Eqs. (9), (28), (18) and (19), we obtain the result. □

4. Numerical results

In this section, we demonstrate the performance of the proposed schemes in terms of transmission efficiency and also the delay. The numerical results provided are obtained from both the analysis and simulation. For comparison, the corresponding results for the available static XOR coding-based (SXC) scheme and dynamic XOR coding-based (DXC) scheme (in Ref. [23]) are also provided.

4.1. Simulation setting

In our simulation of delivering packets from the source to a set of receivers, the packet loss at each receiver R_i follows the Bernoulli distribution. In addition, the packet loss at different receivers is uncorrelated.

For different scenarios with different settings of parameters (like number of receivers M, size of the lost-packet buffer N and link packet loss probabilities), we simulate the multicast transmission of $10,000 \times N$ packets (i.e., one set of packets) by using the non-coding scheme, the available coding-based schemes and also the new schemes, respectively. Since the packet loss probability at each receiver is given, the simulation results do not depend on the locations of receivers. For each scenario, the corresponding average number of transmissions per packet is the ratio of total number of transmissions (used for delivering 10,000 N packets to all receivers) to $10,000 \times N$.

4.2. Transmission bandwidth

The transmission bandwidth of all network coding-based schemes greatly depends on the lost-packet buffer size, so we first investigate the transmission bandwidth of different schemes under different sizes of the lost-packet buffer. Fig. 5 shows the numerical results of different schemes on the transmission bandwidth, where $M = 4$, $p_1 = 0.80$, $p_2 = 0.70$, $p_3 = 0.60$ and $p_4 = 0.50$. For each scheme with a specific N, 20 trials of multicasting $10,000 \times N$ packets are simulated; all the plotted values in Fig. 5 have a 95% confidence interval not larger than 2% of the plotted values. From this figure, we can see that the analytical results on transmission bandwidth nicely

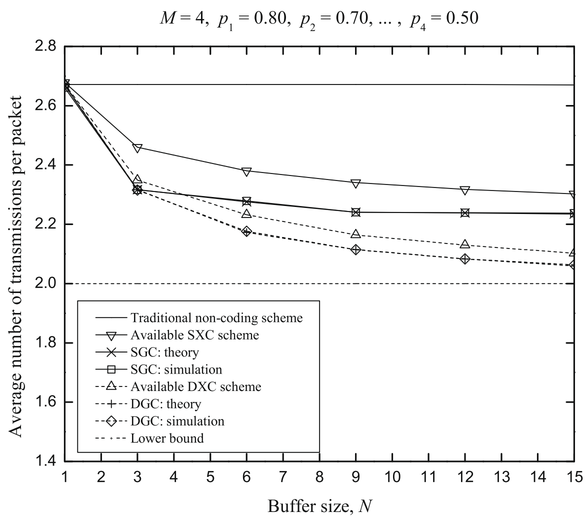

Fig. 5. Transmission bandwidth versus lost-packet buffer size.

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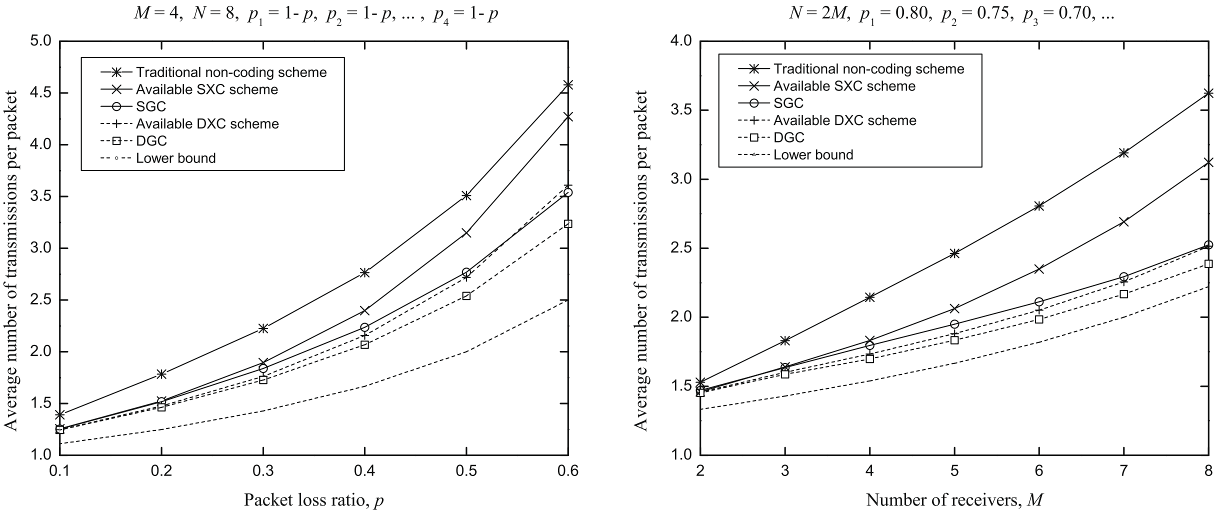

(a) Transmission bandwidth versus packet loss probability

(b) Transmission bandwidth versus number of receivers, in the medium packet loss scenario.

Fig. 6. Transmission bandwidth of different scenarios.

match the simulation results, so the proposed models can be used to efficiently investigate the transmission bandwidth of the proposed schemes. Additionally, we can also observe that in general the transmission bandwidth of each network coding-based scheme decreases as the lost-packet buffer size increases, and when the lost-packet buffer size is not very small, the coding-based multicast schemes can substantially outperform the non-coding multicast scheme. For example, for buffer size $N = 9$, compared to the traditional multicast scheme, the average number of transmissions per packet can be reduced by over 16% when using the proposed static scheme.

From Fig. 5 we can also observe that, compared to the available static scheme, the proposed static scheme can more effectively reduce the transmission bandwidth, especially when the lost-packet buffer size is small. For example, when the buffer size is three, the available static scheme only reduces the bandwidth consumption by 7.9% percent, while this reduction can be 13.3% when using the SGC scheme. Similarly, the proposed dynamic scheme always outperforms the available dynamic scheme. For example, as compared with the available dynamic scheme, the proposed dynamic scheme can further improve the transmission efficiency by 2.2% when $N = 6$. Additionally, results in Fig. 5 show that the dynamic schemes greatly outperform the static schemes, at the cost of increased computational complexity.

Another important conclusion we can draw is that when the buffer size is set to an appropriately large value, the proposed dynamic scheme performs pretty close to the lower bound (shown in Eq. (1)). For example, in Fig. 5, when N is 15, the lower bound is only 2.9% smaller than the efficiency of proposed dynamic scheme. Therefore, the proposed coding-based dynamic scheme is able to achieve quite high transmission efficiency. Similar conclusion can be drawn in other scenarios.

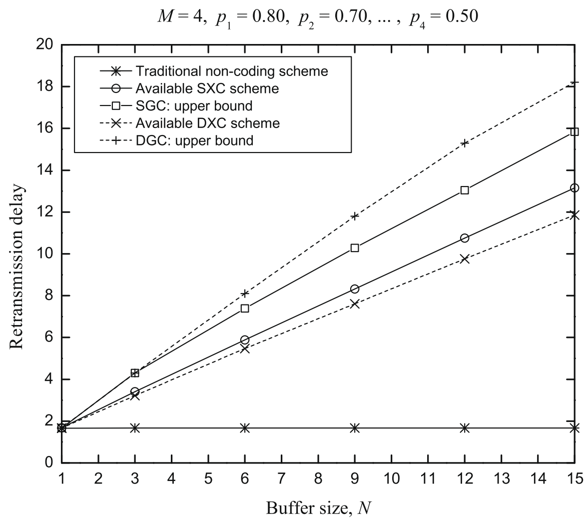

Fig. 7. Retransmission delay versus lost-packet buffer size.

We further investigate the transmission bandwidth under different link packet loss probabilities and different numbers of receivers, as summarized in Fig. 6. The results in Fig. 6(a) show that as the packet loss probabilities increase, the advantage of the proposed schemes over the available schemes becomes more significant. For example, when the packet loss probability of each link is 0.5, compared with the non-coding scheme the available static scheme reduces the bandwidth consumption by 10.3%, but the bandwidth consumption achieved by the SGC scheme can be as high as 21.1%. The results in Fig. 6(b) show that as compared with the available schemes, the transmission bandwidth reduction achieved by using the proposed schemes increases as the number of receivers

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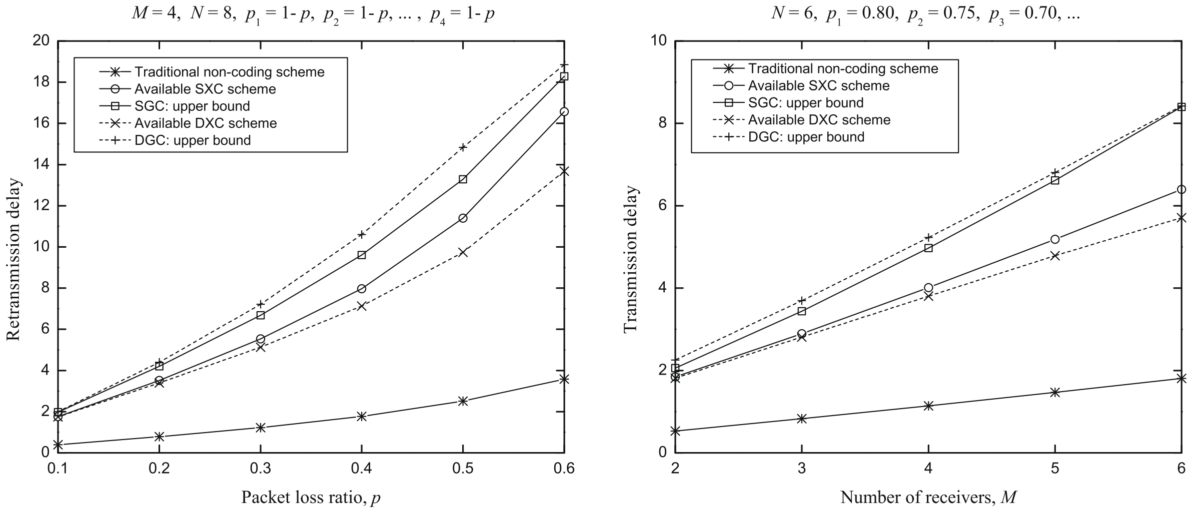

(a) Delay versus packet loss probability. (b) Delay versus number of receivers.

Fig. 8. Retransmission delay of different scenarios.

increases. For example, for the case of three receivers, both the available and the proposed static schemes reduce the transmission bandwidth by about 10.4%. For the case of six receivers, however, the proposed static scheme can reduce the transmission bandwidth by as high as 24.8%, much higher than the 16.3% achieved by the available static scheme.

It should also be noticed that fountain codes can also be applied to the reliable multicast [28–30]. It can be expected that, compared to the fountain code-based scheme, the coding-based schemes will result in a higher transmission efficiency but a larger computational complexity. This can be explained as follows. Let us consider Raptor code, one very good fountain code. To achieve the linear-time encoding and decoding, different from the coding-based schemes which carefully select the packet combination coefficients, Raptor code randomly selects the combination coefficients and then linearly combines the native packets together. The price paid for such a faster encoding and decoding is that, at each receiver, receiving N packets is no longer sufficient to reconstruct a set of N data packets; instead, about 10% extra packets need to be received at each receiver for recovering the whole set of N data packets. On the other hand, network coding-based multicast scheme can achieve higher transmission efficiency than Raptor code-based multicast scheme but has relatively high computational complexity. Thus, both two kinds of schemes have their own advantages and potential applications.

4.3. Retransmission delay

Since the analytical model for the exact delay analysis is not available, the proposed upper bound model is adopted here to roughly demonstrate the delay behavior of the proposed schemes.

Fig. 7 shows the retransmission delay as a function of the lost-packet buffer size, where we can see that the retransmission delay of coding-based schemes approximately linearly increases as the lost-packet buffer size increases. The reason of this behavior is that during the transmission phase the source buffers the lost packets for future packet coding rather than retransmitting them immediately. This delay increment is the cost one needs to pay for acquiring coding opportunities. As discussed previously, the transmission efficiency improvement also steadily increases as the lost-packet buffer size increases. Thus, there is a trade-off between the transmission efficiency and the packet delay when determining the lost-packet buffer size.

From Fig. 7 we can also see that although the upper bounds of the proposed schemes are adopted to compare with the available coding-based schemes, the gap between the new schemes and their corresponding old ones are not big. For example, when the buffer size is 15, the upper bound of retransmission delay of the SGC scheme is only 20.4% larger than the retransmission delay of the old static one.

We further show the retransmission delay under different packet loss probabilities in Fig. 8(a) and the retransmission delay under different number of receivers in Fig. 8(b). A similar conclusion that can be draw from these two figures is that the transmission delay of the proposed coding-based schemes is actually close to that of the old coding-based schemes.

5. Conclusion and future work

In this paper, we have proposed two new network coding-based schemes, which adopt more general coding operation rather than the simple XOR, for reliable link-layer multicast: a static one with low complexity and a dynamic one with relatively higher complexity but better performance. Unlike the available network coding-based schemes which have exponential computational complexity, the proposed schemes run in polynomial time. Moreover, the analytical and simulation results demonstrate

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that, compared with the available coding-based schemes, the proposed schemes can more effectively reduce the bandwidth consumption, especially in the case of high packet loss probabilities and many receivers.

It was also shown that the transmission efficiency improvement from using network coding increases with both the size of lost-packet buffer and also the number of multicast receivers. This improvement can be very significant when the lost-packet buffer size and number of receivers are large enough. E.g., for the case that the number of receivers is six and the buffer size is twelve packets, the transmission efficiency can be improved by as much as 24.8% when the proposed dynamic scheme is adopted. Thus, the network coding provides us a new dimension for a more efficient transmission of reliable link-layer multicast.

Notice that due to the complicatedness of the derived analytical results, it is difficult for us to directly get intuitive conclusions from these results. Thus, it can be the future work to obtain good approximations of these analytical results such that we can easily know the relations between the transmission efficiency and some important parameters like buffer size and the number of receivers. Additionally, in our theoretical analysis, it is assumed that the packet loss at each receiver follows the simple Bernoulli distribution and the packet loss at different receivers is uncorrelated. Therefore, another future work is to examine the transmission efficiency of our schemes under more practical packet loss models. Finally, all the previous and our coding-based reliable multicast schemes do not take the packet delay issue into account. It is interesting and important to further evaluate the delay performance of coding-based schemes, and also extend the current work and design to the delay-guaranteed reliable multicast scheme.

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Appendix A. Proof of Lemma 3

$P(N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M \| S_p| = k)$ will be evaluated by

$$P\left(N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M \| S_p| = k\right) = P\left(N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right) \times P\left(|S_p| = k | N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right) / P(|S_p| = k). \quad (20)$$

Among k packets, the probability that $R_1, \ldots R_M$ fail to receive $i_1, \ldots, i_M$ packets, respectively, is given by

$$P\left[N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right] = \prod_{n=1}^{M} \binom{k}{i_n} p_n^{k-i_n} (1-p_n)^{i_n}. \quad (21)$$

For k packets, the probability that each packet is not correctly received by at least one receiver is given by

$$P(|S_p| = k) = \left(1 - \prod_{n=1}^{M} p_n\right)^k. \quad (22)$$

Now, we will evaluate $P(|S_p| = k | N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M)$. Clearly, the total number of patterns of lost packets, which satisfy that $N_l^{p,n} = i_n, n = 1, \ldots, M$, is $\prod_{n=1}^{M} \binom{k}{i_n}$ (all these patterns happen with the same probability). We proceed to calculate the number N_{epl} of patterns satisfying that $N_l^{p,n} = i_n$ $(n = 1, \ldots, M)$ and each packet is lost at one or more receivers (i.e., $|S_p| = k$). Then

$$N_{epl} = \prod_{n=1}^{M} \binom{k}{i_n} - \left|A_1 \bigcup A_2 \bigcup \cdots \bigcup A_N\right|, \quad (23)$$

where A_i $(i = 1, \ldots, N)$ denotes the set of patterns satisfying that $N_l^{p,n} = i_n$ for $n = 1, \ldots, M$ and packet P_i is received by all receivers.

According to the inclusion-exclusion principle, we have

$$\begin{aligned} &|A_1 \cup A_2 \cup \cdots \cup A_k| \\ &= (|A_1| + |A_2| + \cdots + |A_k|) - (|A_1 \cap A_2| + |A_1 \cap A_3| + \cdots \\ &\quad + |A_{k-1} \cap A_k|) + (|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \cdots \\ &\quad + |A_{k-2} \cap A_{k-1} \cap A_k|) + \cdots + (-1)^{k-1} \times |A_1 \cap A_2 \cap \cdots \cap A_k| \\ &= (-1)^0 \binom{k}{1} \prod_{n=1}^{M} \binom{k-1}{i_n} + (-1)^1 (k2) \\ &\quad \times \prod_{n=1}^{M} \binom{k-2}{i_n} + \cdots + (-1)^{k - \max_{n \in \{1,\ldots,M\}} i_n} \\ &\quad \times \binom{k}{k - \max_{n \in \{1,\ldots,M\}} i_n} \prod_{n=1}^{M} \binom{\max_{n \in \{1,\ldots,M\}} i_n}{i_n} + 0 + \cdots + 0. \end{aligned} \quad (24)$$

Then we have

$$\begin{aligned} &P\left(|S_p| = k | N_l^{p,1} = i_1, \ldots, N_l^{p,M} = i_M\right) \\ &= \left(\prod_{n=1}^{M} (k i_n) - |A_1 \cup A_2 \cup \cdots \cup A_k|\right) / \prod_{n=1}^{M} \binom{k}{i_n} \\ &= \left(\sum_{l=0}^{k - \max_{n \in \{1,\ldots,M\}} i_n} (-1)^l \binom{k}{l} \prod_{n=1}^{M} \binom{k-l}{i_n}\right) \Big/ \prod_{n=1}^{M} \binom{k}{i_n} \end{aligned} \quad (25)$$

Finally, by substituting Eqs. (21), (22) and (25) into Eq. (20), we get the result.

Appendix B. Proof of Theorem 13

The overall retransmission delay D of a generation of packets are induced only by those lost packets, including the waiting time in the transmission phase (TPWT) and the waiting time in the retransmission phase (RPWT). Denote by $P_N, P_{N-1}, \ldots, P_1$ the N transmitted packets in turn. If

P_n is lost, the waiting time of P_n in the transmission phase is $n-1$. Thus D is given by

$$D = \sum_{P_i \in S} i + \sum_{P_i \in S} D_i. \tag{26}$$

where S is the set of lost packets among a generation of packets and D_i is the number of packets transmitted in the retransmission phase until P_i is received by all receivers. Then we have

$$\gamma_g = \frac{1}{N} E[D] = \frac{1}{N}\left(E\left[\sum_{P_i \in S} i\right] + E\left[\sum_{P_i \in S} D_i\right]\right). \tag{27}$$

The term $E[\sum_{P_i \in S} i]$ in Eq. (27) is evaluated by

$$E\left[\sum_{P_i \in S} i\right] = \sum_{L=0}^{N} P[|S| = L] E\left[\sum_{P_i \in S} i \middle| |S| = L\right] = \sum_{L=0}^{N} \binom{N}{L} \left(1 - \prod_{n=1}^{M} p_n\right)^L \left(\prod_{n=1}^{M} p_n\right)^{N-L} \frac{(N-1)L}{2}. \tag{28}$$

In the first equality above, we calculate the average TPWT by considering the conditional average TPWT under different numbers of lost packets. It is easy to obtain that when there are L lost packets, the average TPWT is $(N-1)L/2$.

The term $E[\sum_{P_i \in S} D_i]$ in Eq. (27) is evaluated by

$$E\left[\sum_{P_i \in S} D_i\right] = \sum_{L=0}^{N} \left(P[|S| = L] E\left[\sum_{P_i \in S} D_i \middle| |S| = L\right]\right). \tag{29}$$

During the retransmission of a set S_p of lost packets, in the worst case, each receiver R_i receives $N_l^{p,i}$ retransmission packets exactly after the N_r^p'th retransmission, and recovers each one of $N_l^{p,i}$ lost packets exactly when receiving $N_l^{p,i}$ retransmission packets. Thus, the expected overall delay of lost packets of the n'th set is less than $n \cdot E[N_r^p | |S_p| = M] \cdot M$. Then

$$\begin{aligned} E\left[\sum_{P_i \in S} D_i \middle| |S| = L\right] &< 1 \cdot E[N_r^p | |S_p| = M] \cdot M + \cdots \\ &+ \left\lfloor \frac{L}{M} \right\rfloor \cdot E[N_r^p | |S_p| = M] \cdot M \\ &+ \left(\left\lfloor \frac{L}{M} \right\rfloor E[N_r^p | |S_p| = M] + E[N_r^p | |S_p| = L\%M]\right)(L\%M) = 0.5 E[N_r^p | |S_p| \\ &= M] \left\lfloor \frac{L}{M} \right\rfloor (M + L + L\%M) + E[N_r^p | |S_p| = L\%M](L\%M). \end{aligned} \tag{30}$$

Finally, combining Eqs. (27)–(30) we obtain the result.

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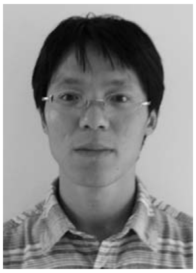

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