Packet Loss Process under Bounded Delay

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***Abstract*— This paper analyzes the loss process distribution of a finite buffer queue. In contrast to the previous work that assumed the buffer can merely store finite number of packets, our model adopts the bounded delay policy where only the packet arrival finding its delay not exceeding a preset value is admitted into the buffer. The quantity of interest is the probability distribution of the number of lost packets within a block of n consecutive packet arrivals, which is an important measure for the design of communication networks, e.g., the forward error correction (FEC). We derive a set of recursive equations to compute the above quantity for various packet size distributions. We then focus on the influence of adding redundant packets on loss probability of message block and FEC efficiency. The impacts of bounded delay, packet size distribution and traffic load are also evaluated. We demonstrate a unique property of the finite queue with bounded delay, which is different from that of the conventional finite queue (e.g., $M/G/1/K$ queue).**

I. Introduction

Packet loss probability is known to be an insufficient measure for the quality of service (QoS) in communication networks. In many applications, the message is often divided into smaller packets which are transmitted consecutively into the network. The loss of a single packet of the message may result in the loss of the whole message. Forward error correction (FEC) techniques are proposed. For example, the erasure recovery codes [1][2][3] can reconstruct up to k losses, by adding k redundant packets to a message. Thus, besides the average packet loss probability, the packet loss process, i.e., the loss probability of j packets out of a message (block) of n consecutive packets, is also a vital quantity for proper design of the real time (e.g., video, voice) coding, playback mechanism and forward error recovery [4][5].

Although the packet loss process has been studied extensively before [4, 6-8], all of the authors confined their investigations to the blocking models where the buffer can merely store finite number of packets. In [4], Cidon *et al.* introduced recursive equations to analyze the above loss probabilities. By assuming Poisson arrival and exponential packet size (length), they obtained the recursive analysis of loss process for the finite $M/M/1/K$ queue. Based on this recursive scheme, Altman *et al.* [6] derived explicit expressions for the multidimensional generating function of the above loss probabilities. In [7], Gurewitz *et al.* revisited the same problem by means of the extensive use of various version of the *Ballot Theory*. In order to analyze the loss process under multimedia streams, Dán *et al.*[8] extended Cidon's recursive scheme to the cases of Markov-modulated Poisson arrival with exponential or deterministic packet sizes.

Two reasons motivated us to study the loss process under bounded packet delay. The first one is that modeling a communication node by a queue with finite buffer, which can hold at most K packets, is usually an approximation, since in most systems the buffer space is measured in bits rather than packets. For a buffer system emptying at a constant rate, an arrived packet is rejected if the sum of its length (in bits) and the lengths of all buffered packets would overflow the buffer capacity. If it can be assumed that the service time of a packet depends only on its size and is thus known upon arrival, then the overall length of the buffered packets is bounded, and the model of a queue with bounded packet delay is more appropriate [9]. Another reason comes from the characteristics of optical buffer [10] in future all-optical networks. Optical buffer delays the packet for some amount of time to avoid output contention, which behaves disparately from the electronic buffer. A packet can be stored in electronic buffer for arbitrary long time, while the maximum delay that a finite optical buffer can provide to the arrived packets is bounded. A packet will be blocked if its required delay exceeds the maximum delay of optical buffer [11-14].

This work considers a finite buffer queue adopting bounded delay policy, where only the packet arrival finding its delay not exceeding a preset value is admitted into the buffer. As the first step, Section II introduces the method to obtain the probability distribution of remaining workload in the buffer under Poisson arrival and general packet size distribution. And then in Section III, recursive equations are developed based on the remaining workload distribution to compute the quantities of loss process. From the loss process distribution, we get several useful measures, e.g., packet loss probability, loss probability of message block, FEC gain and average packet loss burst size. Extensive numerical results are presented in Section IV. We will focus on the influence of adding redundant packets on the loss probability of message block and FEC gain. We also demonstrate the impacts of bounded delay, packet size distribution and traffic load on the above performance measures. In addition, it is worth noting that unlike the conventional finite queue (e.g., $M/G/1/K$) where a bigger variance of packet size distribution results in larger queueing and hence larger loss probability [15], the finite queue under bounded delay shows some unique characteristics.

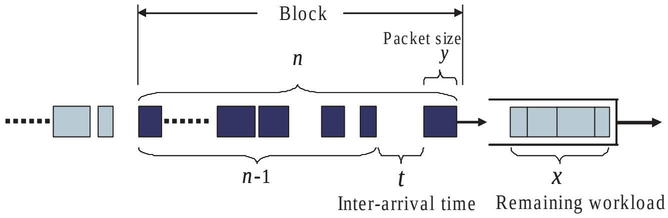


Fig. 1. Queueing model of the packet loss process

II. The finite buffer queue with bounded delay

We consider a finite buffer queue with single output link as depicted in Fig.1. Packets arrive at the system according to Poisson process with rate λ and are served (transmitted) with the first-in-first-out (FIFO) policy. For the convenience of analysis, the packet size is measured in terms of transmission time, following a general distribution with probability density function (PDF) $s(y)$ and mean $1/\mu$. The traffic load is then defined as $\rho \equiv \lambda/\mu$.

Each packet arrival is characterized by its service time requirement y and its waiting time x for those packets ahead of it, i.e.,the remaining workload seen in the buffer. The delay of a packet is defined as $D = x + y$. The delay constraint is defined as:

For an arrived packet, if $D \leq T$, it joins the buffer and remains until transmitted; if $D > T$, it is blocked and lost.

The packets are grouped into fixed size blocks, namely, every n consecutive packets form a *block* and we are interested in the probability distribution of the number of lost packets within a block in steady state.

A. Distribution of remaining workload

As a packet arrival is admitted or not according to its size and the remaining workload in the buffer, we first need to derive the distribution $w(x)$ of remaining workload before we analyze the packet loss process.

Suppose that in steady state, with probability Q the system is empty (i.e., remaining workload $x = 0$), then from the method introduced in [16], we can derive $w(x)$ with the following procedures.

First, under bounded delay policy, the remaining workload has $0 \leq x \leq T$. In this finite queue, $w(x)$ and Q satisfy the normalization condition

$$1 = Q + \int_{0^+}^{T} w(x)dx. \tag{1}$$

And then if we set

$$w(x) = Qh(x), \quad 0 \leq x \leq T, \tag{2}$$

$h(x)$ can be derived by resolving the following integro-differential equation set [16]

$$\begin{cases} \dfrac{dh(x)}{dx} + \lambda s(x) - \lambda \left[\displaystyle\int_0^{T-x} s(u)du \right] h(x) \\ \qquad\qquad + \lambda \displaystyle\int_0^{x} s(x-u)h(u)du = 0 \\ h(0) = \lambda \displaystyle\int_0^{T} s(u)du. \end{cases} \tag{3}$$

While Q can be obtained from (1) as

$$Q = \left[1 + \int_{0^+}^{T} h(x)dx \right]^{-1}. \tag{4}$$

Usually, we cannot resolve (3) analytically for a general packet size distribution $s(y)$. However, numerical solution is always feasible, since even a fine quantization of (3) is not expected to consume excessive computer time.

We can compute the *average packet loss probability* based on the packet size y and remaining workload x as

$$\begin{aligned} P_{loss} &= 1 - Prob\left[D \leq T\right] \\ &= 1 - QProb\left[y \leq T\right] - \int_0^T w(x)Prob\left[y \leq T - x\right]dx. \end{aligned} \tag{5}$$

B. Closed form results for $M/M/1$ queue

When packet size is exponentially distributed,

$$s(y) = \mu e^{-\mu y}, \quad y > 0. \tag{6}$$

Equation (3) can be resolved analytically, getting the closed-form formula for $w(x)$ and P_{loss}.

For such an $M/M/1$ queue with bounded delay T, we have

$$Q = (1-\rho)\left[1 - \rho a + \sum_{n=1}^{\infty} \frac{\rho^n (b^n - a)}{(\rho)_n}\right]^{-1}, \rho \neq 1, \tag{7}$$

$$h(x) = \lambda b^{1-\rho}(z^{\rho-1} - z^{\rho})e^{\rho b - \rho z}, \qquad 0 \leq x \leq T, \tag{8}$$

$$P_{loss} = Qb^{1-\rho}e^{\rho(b-1)}, \tag{9}$$

where

$$h(0) = \lambda(1-b), \quad b \equiv e^{-\mu T}, \quad z \equiv be^{\mu x}$$

$$a \equiv b^{1-\rho}e^{\rho(b-1)}, (\rho)_n \equiv \rho(\rho+1)(\rho+2)...(\rho+n-1).$$

III. Loss process under bounded delay

From the remaining workload distribution $w(x)$ derived above, we can compute the probabilities $P(j,n)$ of j losses in a block of n consecutive packets, $n \geq 1, 0 \leq j \leq n$. Since the first packet in a block is arbitrary, we have the relation

$$P(j,n) = \int_0^T P_x(j,n)w(x)dx, \tag{10}$$

where $P_x(j,n), 0 \leq x \leq T$ is the probabilities of j losses in a block of n packets, given that the remaining workload in the buffer is x at the arrival epoch of the first packet in the block.

Noting that the block structure is an indexing imposed on the sequence of Poisson arrivals starting at an arbitrary location, the queueing behavior is hence intact [7].

A. The recursive equations of $P_x(j,n)$

We develop the following recursive scheme to calculate the the conditional probabilities $P_x(j,n), 0 \le x \le T, n \ge 1, 0 \le j \le n$. Let us first denote by $a(t) = \lambda e^{-\lambda t}$ the inter-arrival time PDF of Poisson process.

By observing Fig.1, the recursion is initiated from $n = 1$ with the following relation for $0 \le x \le T$

$$P_x(j,1) = \begin{cases} \int_0^{T-x} s(y)dy & j = 0 \\ \int_{T-x}^{\infty} s(y)dy & j = 1 \\ 0 & j \ge 2, \end{cases} \tag{11}$$

The physical meaning of (11) is quite clear: if the message block consists of only one packet ($n = 1$) and the remaining workload in the buffer is x, an arrived packet of size y is admitted ($j = 0$) when $y \le T - x$; rejected ($j = 1$), otherwise.

When $j = 0, n \ge 2$, we have

$$\begin{aligned} P_x(0,n) = &\int_0^{x+y} \left[\int_0^{T-x} P_{x+y-t}(0,n-1)s(y)dy \right] a(t)dt \\ &+ \int_{x+y}^{\infty} \left[\int_0^{T-x} P_0(0,n-1)s(y)dy \right] a(t)dt. \end{aligned} \tag{12}$$

Equation (12) describes the case of no loss in a message block consisting of n packets. If the first packet of the block faces the remaining workload x, because all the packets in the block are admitted, the size y of first packet must satisfy $0 < y \le T - x$, corresponding to the integrations in the square brackets. Then the rest of this block can be regarded as a new block with length $n - 1$ (referred to Fig.1), and its first packet will see:

1) remaining workload $x + y - t$, provided that the inter-arrival time $t < x + y$, corresponding to the first term of (12);
2) remaining workload 0, provided that the inter-arrival time $t \ge x + y$, corresponding to the second term of (12).

When $j \ge 1, n \ge 2$, we have

$$\begin{aligned} P_x(j,n) = &\int_0^{x+y} \left[\int_0^{T-x} P_{x+y-t}(j,n-1)s(y)dy \right] a(t)dt \\ &+ \int_{x+y}^{\infty} \left[\int_0^{T-x} P_0(j,n-1)s(y)dy \right] a(t)dt \\ &+ \int_0^{x} \left[\int_{T-x}^{\infty} P_{x-t}(j-1,n-1)s(y)dy \right] a(t)dt \\ &+ \int_x^{\infty} \left[\int_{T-x}^{\infty} P_0(j-1,n-1)s(y)dy \right] a(t)dt \end{aligned} \tag{13}$$

Equation (13) is the case of having loss in a block. Given j losses in the block with n packets, we have two scenarios:

1) the first packet of the block is admitted, then the j losses must happen in the rest $n - 1$ packets, corresponding to the first two terms in (13);
2) the first packet of the block is lost, then only $j - 1$ losses happen in the rest $n - 1$ packets, corresponding to the last two terms in (13).

For Scenario 1, following a similar idea as equation (12) we can get the first two double integrals. For Scenario 2, since the first packet is lost, if we consider the rest $n - 1$ packets as a new block, its first packet will only face remaining workload $x - t$ or 0, depending on the inter-arrival time t (referred to Fig.1).

Equations (11), (12) and (13) form the base to compute $P_x(j,n), 0 \le x \le T, n \ge 1, 0 \le j \le n$, and then we can apply equation (10) to get the packet loss distribution $P(j,n), n \ge 1, 0 \le j \le n$, finally.

B. Loss probability of message block and FEC gain

Based on the loss process distribution, we can obtain several useful performance measures. For example, under Poisson arrival, the *average packet loss probability* (5) can be calculated from the packet loss within the message block in steady state as

$$P_{loss} = \frac{\sum_{j=0}^{n} jP(j,n)}{n}. \tag{14}$$

To reduce the overall loss of message blocks, the FEC techniques add redundancy to the transmitted stream and recover losses based on redundant information. For instance, the widely used block coding scheme like Reed-Solomon coding [2], can have $k(0 \le k < n)$ redundant packets and $n - k$ data packets in each block, we denote such a scheme as $FEC(n-k,n)$. If the loss is not more than k packets, the message block can be recovered. For a message block with n packets, when no redundant packet is added, *loss probability of message block* is

$$M_{loss} = 1 - P(0,n). \tag{15}$$

When applying a coding scheme $FEC(n-k,n)$ to the block with n packets, message block loss probability becomes

$$M_{loss}^{FEC} = 1 - \sum_{j=0}^{k} P(j,n) = \sum_{j=k+1}^{n} P(j,n). \tag{16}$$

For the above coding scheme, if more than k packets are lost in a block, the lost packets cannot be corrected, and the *uncorrected packet loss probability* can be computed as

$$P_{loss}^{FEC} = \frac{\sum_{j=k+1}^{n} jP(j,n)}{n}. \tag{17}$$

To evaluate the efficiency of an FEC scheme, we define the *FEC gain* as

$$G_{FEC} = \frac{P_{loss}}{P_{loss}^{FEC}}. \tag{18}$$

C. Average loss burst size

It is notable that the packet losses are bursty in nature and show strong correlation [4]. Many applications (e.g. video, voice and the error recovery techniques) are sensitive to long bursts of packet loss, which may significantly degrade the QoS. In our case, the *average loss burst size* $E[B]$, i.e., the average for the number of consecutively lost packets B, can be calculated as [8]

$$E[B] = \frac{\sum_{n=1}^{\infty} P(n,n)}{P_{loss}}. \quad (19)$$

IV. Numerical results and discussions

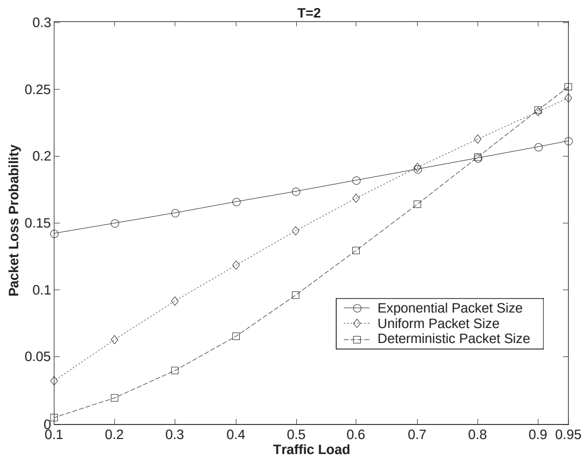
This section numerically evaluates the characteristics of loss process and forward error correction under bounded delay. In our analysis, we will focus on the following PSDs: *exponential*, *uniform* and *deterministic*, and they all have the same mean packet length of $1/\mu$. Times are normalized by $1/\mu$ such that $\lambda = \rho$. In all figures displayed in the sequel, simulation results are marked by $\diamond, \circ$, etc., and numerical results are designated by solid or dash lines.

Fig. 2a and Fig. 2b present the packet loss probability (5) and average packet loss burst size (19) for different traffic load and a bounded delay $T = 2$. As traffic load increases, both the loss probability and loss burst size increase accordingly.

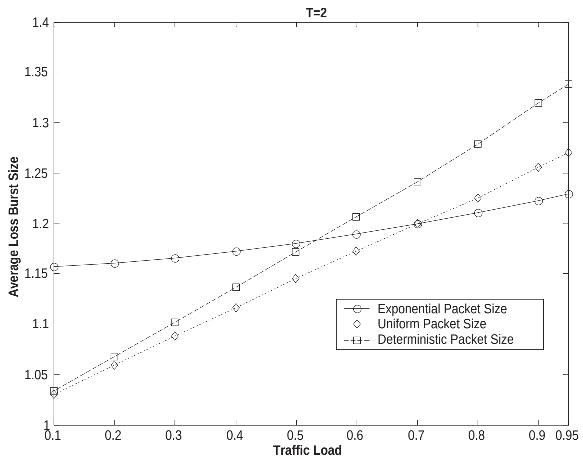
It is well known that a bigger PSD variance leads to larger queueing and hence larger packet loss [15] in the conventional finite queue (e.g., the $M/G/1/K$ queue). However, for the finite queue under bounded delay, although the exponential PSD with the largest variance results in the largest packet loss for relatively small ρ (e.g., $\rho < 0.7$ in Fig. 2a), the scenario is just the opposite under heavy load. We can see that the deterministic PSD with the smallest variance will lead to the largest loss probability when $\rho > 0.9$ in Fig. 2a.

Comparing Fig. 2b with Fig. 2a, we also observed that a bigger packet loss probability leads to bigger average loss burst size. The influence of PSD on loss burst size is similar with that on packet loss probability: when ρ is relatively small (e.g., < 0.5 in Fig. 2b), the exponential PSD suffers the largest loss burst size; while $\rho > 0.7$, it will have the smallest loss burst size.

Fig. 3a shows the effect of redundant packets on the loss probability of message block, when $T = 10, \rho = 0.8$, and each message block consists of 20 packets among which the number of redundant packet k varies from 0 to 8. We can see that the increasing of redundant packet effectively decreases the block loss probability, while also reduces the number of useful data packets. Therefore, a careful trade-off must be examined when designing the FEC mechanism. Although the exponential PSD faces the largest block loss probability for small k (e.g., $k < 4$ in Fig. 3a), it benefits the most from adding redundant packets, since further increasing the redundant packets will make it have the smallest block loss. For the deterministic PSD, the scenario is just the opposite.



(a)Packet loss probability vs. Traffic load when T=2



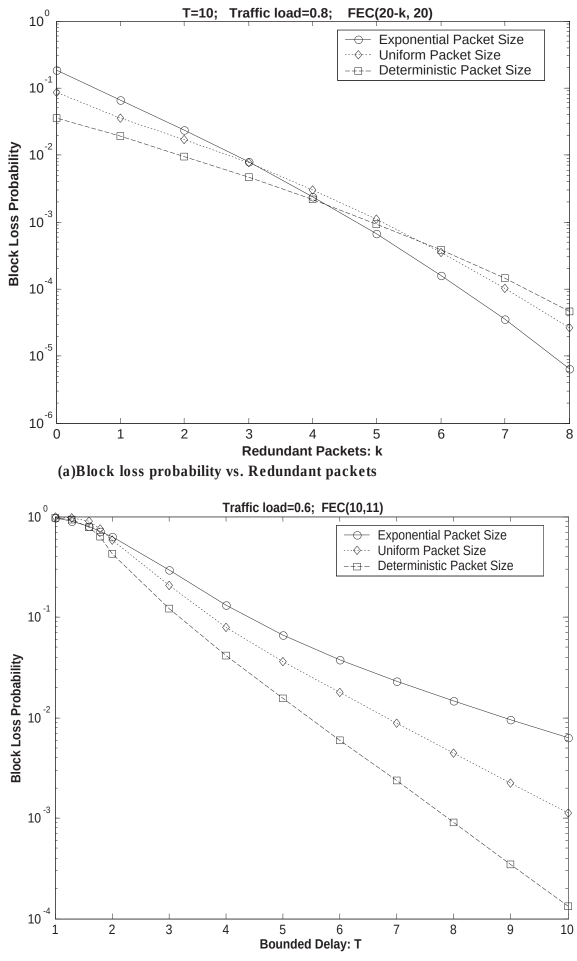
(b)Average loss burst size vs. Traffic load when T=2

Fig. 2. (Packet loss probability, Average loss burst size) Vs. Traffic load

Fig. 3b evaluates the impact of bounded delay T on the loss probability of message block when $\rho = 0.6$. Here we adopt the coding scheme $FEC(10, 11)$, i.e., there is only one redundant packet in each message block. We can see that enlarging the bounded delay reduces the block loss effectively, and the exponential PSD has the biggest block loss, since it suffers the largest packet loss for its biggest variance when ρ is relatively small.

Fig. 4a and Fig. 4b demonstrate the influence of bounded delay on FEC gain G_{FEC} when $\rho = 0.8$ and $\rho = 0.4$ under coding scheme $FEC(10, 11)$. We have observed that as T increases, G_{FEC} will converge and the exponential PSD will have the largest FEC gain for its largest variance.

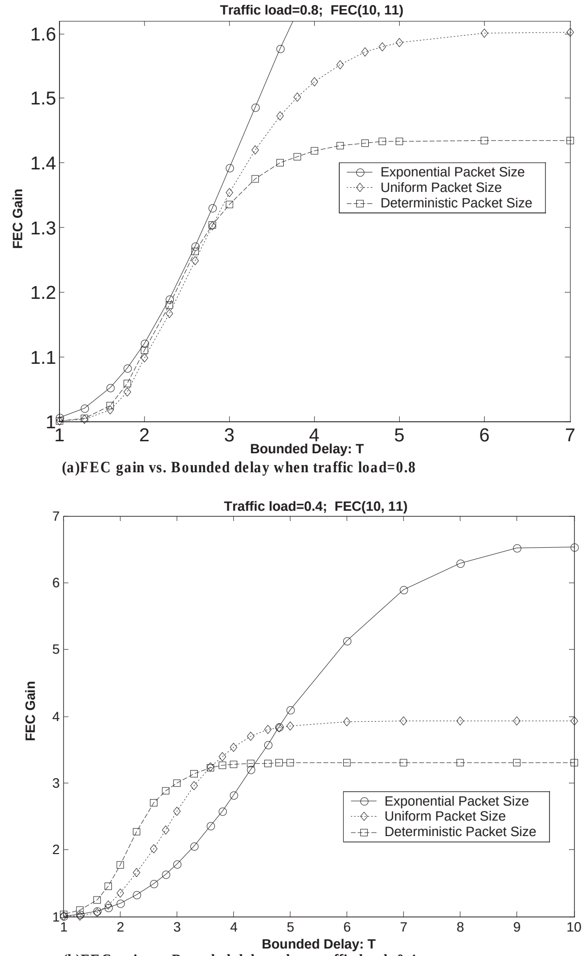
Fig. 5a and Fig. 5b demonstrate the influence of traffic load on FEC gain when $T = 5$, $T = 2$, respectively. We have observed that as ρ increases, G_{FEC} will converge (to 1) and the exponential PSD will have the largest FEC gain again. Combining Fig. 5b with Fig. 2b, when $T = 2$, we can also see that a bigger average loss burst size contributes to the lower efficiency of FEC. For example, when ρ is relatively small (e.g., < 0.5), the exponential PSD has the biggest average loss



(a)Block loss probability vs. Redundant packets

(b)Block loss probability vs. Bounded delay

Fig. 3. Block loss probability Vs. (Redundant packets, Bounded delay)



(a)FEC gain vs. Bounded delay when traffic load=0.8

(b)FEC gain vs. Bounded delay when traffic load=0.4

Fig. 4. FEC gain Vs. Bounded delay under different traffic load

burst size in Fig. 2b, which leads to its lowest FEC gain in Fig. 5b. However, from Fig. 3a, 4 and 5, we can still conclude that forward error correction is generally more effective and more efficient for the PSD with bigger variance provided that bounded delay is relatively large (e.g., $T > 5$).

V. Conclusions and Future Works

We have analyzed the packet loss process for a finite buffer queue under bounded delay, which has different characteristics from the conventional finite queue (e.g., $M/G/1/K$). First, we introduced the method to derive the remaining workload distribution and packet loss probability for our finite model. And then we developed a recursive scheme to compute the probability distribution of the number of lost packets within a message block of n consecutive packets, from which we obtained the average loss burst size, and evaluated the effect of adding redundant packets on the loss probability of message block. Through extensive numerical results, we further demonstrated the impact of bounded delay, packet size distribution and traffic load on the FEC efficiency.

One of our future work is to incorporate more complex traffic patterns into our problem, e.g., the Markov-modulated Poisson process (MMPP), or Markov-modulated rate process (MMRP), which can capture the characteristics of versatile traffic sources. Another direction is to examine the loss process under multi-hop scenario, which is more realistic and hence valuable for the design of error correction.

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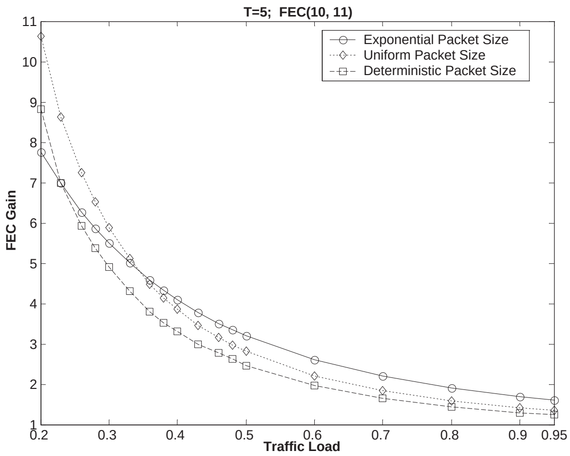
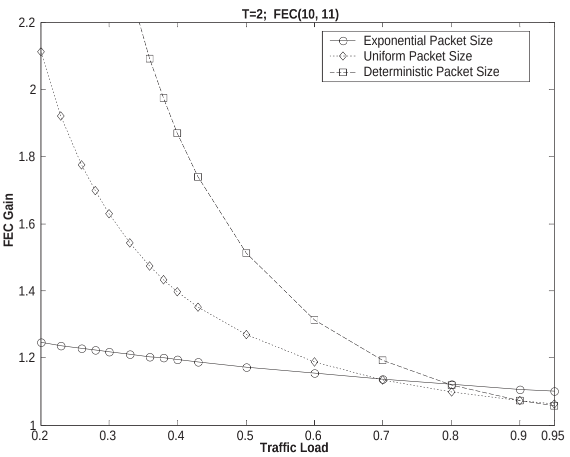


(a)FEC gain vs. Traffic load when T=5



(b)FEC gain vs. Traffic load when T=2

Fig. 5. FEC gain Vs. Traffic load under different bounded delay

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