Pruned Optical Banyan Networks on Vertical Stacking Scheme for Faster Connection Establishment

Md. Mamun-ur-Rashid Khandker, Xiaohong Jiang, Masaru Fukushi and Susumu Horiguchi
Graduate School of Information Sciences, Tohoku University, Sendai, Japan.
{khandker, susumu, jiang, mfukushi}@ecei.tohoku.ac.jp

Abstract

In this paper we address the issue of faster connection establishment in a large vertically stacked optical Banyan (VSOB) network. The best known global routing algorithm, which turns an $N \times N$ crosstalk-free VSOB network into a rearrangeably nonblocking one, has time complexity $O(N\log N)$. This is quite large compared to $O(\log N)$ time complexity of a single plane banyan network, which is a self-routing network with very high blocking probability. For a large size of switching network this $O(N\log N)$ time complexity may result unacceptably long delay. Therefore, an optical network with very low blocking probability and $O(\log N)$ time complexity will be useful. Previously proposed Plane Fixed Routing (PFR) algorithm has $O(\log N)$ time complexity but results in higher than 2% blocking probability with zero-crosstalk constraint for a network as large as 4096×4096 at full load. In this paper, first we propose the pruning of VSOB networks that reduces the hardware cost by almost 30%. The networks can still use the PFR algorithm and results in the same blocking probability. However, we show that the blocking probability can be reduced dramatically while keeping the optimum time complexity $O(\log N)$ by allowing only a small amount of crosstalk. Then, we propose a new kind of switching networks in which extra regular banyan planes have been added with the pruned VBOS (P-VSOB) networks. Necessary routing algorithms, namely, PFR_RS and PFR_LS show that this new switching network can reduce the blocking probability to very low value even with zero-crosstalk constraint while keeping the hardware cost almost the same as for P-VSOB networks. Both these algorithms also have time complexity $O(\log N)$.

Key words: Vertically stacked banyan networks, Crosstalk, Rearrangeable non-blocking, Self-routing, Plane Fixed Routing algorithm.

I. INTRODUCTION

Photonic switching networks play an important role within wavelength-division multiplexing (WDM) networks, where optical cross-connects (OXCs) are greatly preferable to electronic switches. The main factors those have to be considered while designing any photonic switching networks are hardware cost, blocking probability, crosstalk, switching speed etc. Directional-coupler (DC) [1] can handle optical signals of some terabits per second and with multiple wavelengths, and this makes it ideal for serving as the basic 2×2 switching element (SE) in high-speed optical switches. However, DC suffers from an intrinsic crosstalk problem [1][2], in which a portion of optical power in one waveguide of a DC will be coupled into the other waveguide unintentionally when two optical flows pass through the DC at the same time. This undesirable coupling effect is called first-order crosstalk, which may propagate downstream stage by stage, leading to a higher order crosstalk in each downstream stage with a decreasing magnitude. In this paper we will consider only first-order SE crosstalk, and it is therefore understood that in this paper the term "crosstalk is used as an abbreviation for "first-order crosstalk." Widely used crossbar architecture cannot be used for switching networks larger than 32x32 in which a signal may have to pass through 63 crosstalk SEs (hereafter CSEs). A cost-effective solution to the crosstalk problem is to make sure that only one signal passes through a DC at a time such that the first-order crosstalk can be eliminated.

Banyan networks [3][4] are attractive for constructing DC-based optical switches for their small depth, absolute loss uniformity (each path goes through exactly the same number of DCs), lower switch count (lower hardware cost) and a simple switch setting ability (self-routing). With banyan structure only a unique path exists between an input-output pair, in which the network is degraded as a blocking one. As shown in [5][6], the blocking probability of banyan networks is usually very high. For example, the blocking probability for a 256×256 banyan network can be as high as 0.65 (at full load condition), even without any crosstalk constraint. To deal with this situation, it is a novel approach to keeping the whole network nonblocking as well as crosstalk-free by vertical stacking multiple copies (planes) of an optical banyan network [7]. This network is called vertically stacked optical banyan (VSOB) network, which preserves the nonblocking characteristic while neither increasing the number of stages nor sacrificing the loss uniformity property intrinsically possessed by the banyan network structures. A VSOB network can be rearrangeably nonblocking with a routing algorithm [8][9] having time complexity O(Nlog$N$). However, in most practical cases, switching networks with very low blocking probability and crosstalk can do the work. In this paper, all logarithms use 2 as the base.

A rearrangeably nonblocking VSOB network is not self-routing network because it requires a global routing algorithm to distribute connections properly among its planes such that all connections can be realized in a nonblocking (and crosstalk-free) way. The best known routing algorithm for a rearrangeable N×N VSOB network requires a time complexity O(Nlog$N$) [8][9], which is significantly higher than the optimum routing complexity O(log$N$) of self-routing banyan networks. Khandker et. al. [10] has proposed PFR routing algorithm in which a plane of the VSOB network is fixed for a group of inputs and achieved $O(\log N)$ time complexity. However, the blocking probability is always more than 2% even for a network as large as 4096×4096 at full load (port occupancy probability r has been considered as the load). Therefore, a switching network with optimum time complexity $O(\log N)$ and very low blocking probability is sought.

In this paper, first we show that allowing only 3 CSEs along any signal path the blocking probability can be drastically reduced (<0.0004% at $r = 1.0$) by the PFR algorithm. The results are quite impressive considering its time complexity $O(\log N)$. We also propose pruning of each banyan plane which reduces the hardware cost almost by 30%. Actually, the PFR algorithm has turned the pruned VSOB network into a self-routing network, so far the first of this kind. Considering that zero crosstalk constraint may be required in some special cases, we propose Extended Pruned VSOB network (here after EP-VSOB), in which we add small number (only 2 or 3) of regular banyan planes with the P-VSOB network. Necessary routing algorithms provided with this switching network results in very low blocking probability (<0.03%) even with zero-crosstalk constraint.

The rest of the paper is organized as follows. We briefly introduce the VSOB network in Section II, and then we discuss the PFR algorithm for the VSOB network in Section III. In section IV we propose the construction of the P-VSOB networks that can also use the plane fixed routing (PFR) algorithm. Section V describes the EP-VSOB network architecture and two new routing algorithms for it. Section VI deals with the performance analysis of the new algorithms and the switch architecture, and Section VII summarizes the contributions of the paper.

II. VSOB NETWORK

An $N \times N$ VSOB network is constructed by vertically stacking multiple copies (planes) of an $N \times N$ optical banyan network, with each SE being a DC. Since only one signal is allowed to pass through an SE at a time to guarantee the crosstalk-free property, thus only a few connections can be established in one optical banyan network. To accommodate all N connections without blocking, the number of planes of a VSOB network must meet some requirements. The following theorem gives the condition of rearrangeably nonblocking VSOB network [8][9].

Theorem1: An $N \times N$ VSOB network is rearrangeably nonblocking and crosstalk-free if and only if its number of planes T meets the following condition:

$$T = 2^{\lfloor (\log N+1)/2 \rfloor} \tag{1}$$

The rearrangeably nonblocking VSOB network has much lower hardware cost than that of the strictly nonblocking VSOB network [11] and the wide-sense nonblocking VSOB network [12], and the rearrangeably nonblocking VSOB is actually optimal in the sense that it consists of the minimum number of planes required by a VSOB network to be crosstalk-free. Hereafter, we use VSOB(N,T) to refer to the rearrangeably nonblocking $N \times N$ VSOB network with its number of planes T being determined by Eq.1.

To realize any permutation in a VSOB(N,T) network, a global routing algorithm must be adopted to distribute connections properly among its planes such that all connections can be realized in a nonblocking way. The best known routing algorithm for realizing a full permutation in VSOB(N,T) network requires a time complexity O(Nlog$N$) [8][9]. The main idea of the routing algorithm is to decompose a permutation evenly into T crosstalk-free realizable partial permutations (CRPPs) and realize each CRPP in a stacked plane of the network. Some input-output hardware is also required. At the input side, N 1:T switches are required to send a signal to only one plane in a routing phase, and at the output side N T:1 combiners are required, but only one of T links of a combiner will be active at a time. It is better to mention that even a distributed control exhaustive search algorithm that has $O(\sqrt{N}\log N)$ time complexity cannot achieve zero blocking probability.

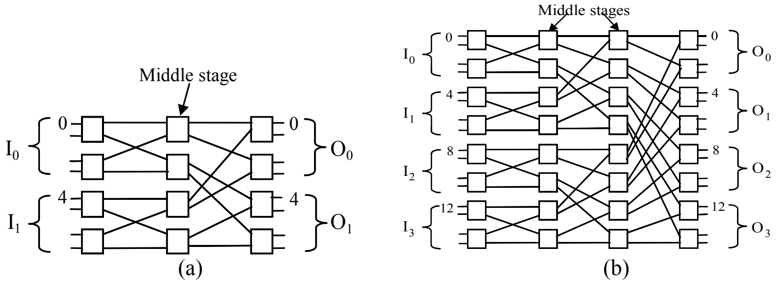
III. PLANE FIXED ROUTING (PFR) ALGORITHM FOR VSOB(N,T) NETWORK

In this section, we first introduce the inputs (outputs) grouping of banyan networks proposed in [8][9], and then describe the fast routing algorithm, namely PFR, for VSOB(N,T) network based on the idea of inputs grouping.

A. Grouping of Inputs and Outputs

For an $N \times N$ banyan network, we use the parameter T in Eq.1 to group its input set I={0,1,…,N-1} (output set O={0,1,…,N-1}) into the following disjoint groups I_i (O_i) as illustrated in Figure 1.

$$I_i = O_i = \{i \cdot T, i \cdot T + 1, \ldots, i \cdot T + (T-1)\},\ \ 0 \le i \le \frac{N}{T} - 1 \tag{2}$$



(a) (b)

Figure 1: Illustration of inputs and outputs grouping. (a) Network with odd number of stages. (b) Network with even number of stages.

Then we have the following Lemma regarding the property of the inputs grouping in Eq.2 [8][9].

Lemma 1: Any two connections in an $N \times N$ banyan network will be SE-disjoint from the first stage up to at least the middle stage if their inputs fall within two distinct groups in Eq.2.

It is clear from Lemma 1 that if all inputs of a set of connections are group-disjoint, then this set of connections will be crosstalk-free (nonblocking) in, at least, the first half stages of an optical banyan network. Based on this property of inputs grouping, the Plane Fixed Routing algorithm for VSOB(N,T) networks has been proposed.

B. Plane Fixed Routing (PFR) algorithm

In the PFR algorithm, we select one input from each input group and tie them with a plane of VSOB(N,T) network permanently. Since all the inputs tied with a plane are group disjoint, the crosstalk-free property of Lemma 1 is guaranteed. One possible plane assignment of PFR algorithm can be performed as following. We first define the following subsets of input set I={0,1,…,N-1}:

$$g_i = \left\{ i, i+T, i+2T, \ldots, i + \left(\frac{N}{T} - 1 \right) T \right\},\ \ 0 \le i \le T - 1 \tag{3}$$

We then tie all inputs in set g_i to the plane i of the VSOB(N,T) network (suppose the planes are numbered as 0,1,…,T-1). For example, the subsets g_i ($0 \le i \le 3$) for VSOB(16,4) network are determined as g_0={0,4,8,12}, g_1={1,5,9,13}, g_2={2,6,10,14}, and g_3={3,7,11,15}, and the final plane assignment of inputs is illustrated in Figure 4.

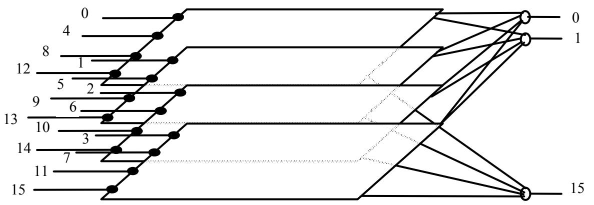


Figure 2: Plane assignment for inputs in VSOB(16,4) network employing PFR algorithm.

A switching network is said to be self-routing if each input-signal can determine its path to the destined output by its destination address, regardless of the destination addresses of other connections. Since each plane of VSOB(N,T) is a self-routing banyan network[3],[4],[13], the VSOB(N,T) network employing PFR scheme is actually a self-routing network because there exists only a unique path between each input-output pair in such network. Therefore, a VSOB(N,T) network employing PFR scheme has the routing time complexity of O(log$N$), i.e. proportional to number of stages of the network, and this time complexity is optimum because we need at least O(log$N$) time to establish a path in a log$N$ – stage banyan network.

It is notable that, the PFR algorithm can only guarantee the nonblocking (crosstalk-free) property in the first half stages of the network, and blocking may occur in the last half stages.

IV. PRUNED-VSOB OR P-VSOB(*N*,*T*) NETWORKS

It can be seen easily from Figure 2 that in PFR algorithm all N inputs are always evenly distributed among the planes as per Eq.3, and each plane has only N/T connections. Since these N/T connections are fixed at their corresponding inputs in the plane, only 1 input of a group (as defined by Eq.2 in Sec. III(A)) will be active; all other inputs will remain unused. Therefore, input switches connected to unused inputs and switches in the successive stages corresponding to these unused inputs are redundant. All these redundant switching elements are eliminated. Figure 3 explains the idea.

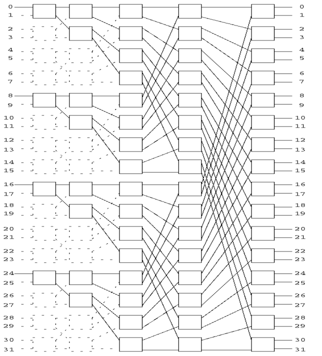


Figure 3: Illustration of pruning of a Banyan network. Redundant SEs are drawn in dashed lines.

V. EXTENDED P-VSOB(*N*,*T*+*K*) NETWORK

Although PFR scheme guarantees very low blocking probability (as it will be shown in Section VI) allowing only 3 crosstalk SEs along a signal path, the blocking probability is always larger than 0.02 with zero-crosstalk-constraint, even for a network as large as 2048×2048. For some high performance applications, where zero-crosstalk-constraint is to be ensured, this blocking probability may be considered too high. In this section, we propose a new network architecture based on the P-VSOB(N,T) network, namely Extended-Pruned-VSOB network, that has the potential to reduce the blocking probability dramatically while keeping the crosstalk constraint to zero, and then we propose necessary routing algorithms modifying the PFR algorithm for faster connection establishment in the new architecture.

A. Extended Pruned VSOB(N,T+K)

The main idea of the extended-pruned-VSOB network is to add a small number of extra planes to the P-VSOB(N,T) network, such that the connections blocked in the fixed T planes of the P-VSOB(N,T) network have more chances to be established through these extra planes. We use EP-VSOB(N,T+K) to denote an extended pruned VSOB network with a K number of extra planes. In the EP-VSOB(N,T+K) network, each input is connected to its fixed plane in the same way as it is in the P-VSOB(N,T) network employing PFR algorithm, and is also connected to the *K* extra planes by a 1:(*K+1)* switch. Figure 4 illustrates the architecture of the EP-VSOB(16,4+2) network.

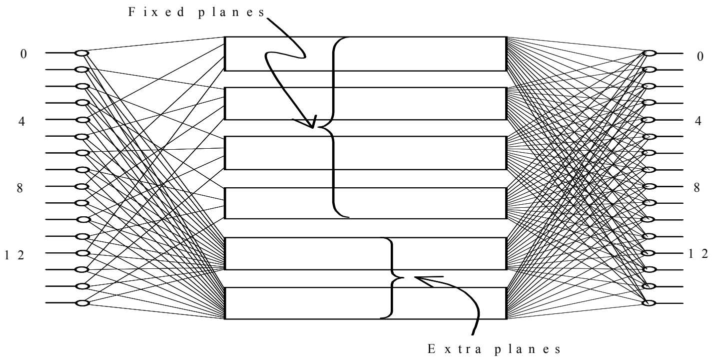


Figure 4: The structure of a EP-VSOB(16,4+2) network.

We will show in the next section that by adopting a suitable routing algorithm, the EP-VSOB(*N*,*T*+*K*) network has the potential to achieve very low blocking probability with only a small number of extra planes and the optimum time complexity *O*(log*N*).

B. Fast Routing Algorithms for EP-VSOB(N,T+K) Network

Depending on how a plane is chosen from the extra planes, we propose two routing algorithms for fast connection establishment in an EP-VSOB(*N*,*T*+*K*) network, namely PFR with random selection algorithm and PFR with linear search algorithm.

1) PFR with Random Selection (PFR_RS)

In the PFR_RS algorithm for an EP-VSOB(*N*,*T*+*K*) network, each connection has two chances to be established through the network. The first chance is to establish the connection through its fixed plane. If the connection request is blocked in its fixed plane, it still has a second chance to be established through another plane selected randomly from the *K* extra planes. If the connection request still could not be established through the randomly selected plane, it is considered as a blocked request. It is easy to see that the time complexity of the PFR_RS algorithm remains the optimum *O*(log*N*).

2) PFR with Linear Search (PFR_LS)

In the PFR_LS algorithm for an EP-VSOB(*N*,*T*+*K*) network, each connection now has (1+*K*) chances to be established through the network. Whenever a request arrives at an input, first it is sent to the fixed plane of that input. If the request cannot be established in the fixed plane, the input searches for a free plane among the extra *K* planes. The searching starts with the first plane, and continues orderly up to the last one so that each plane is being searched at most one time by an input. If the connection request still fails to find a free plane among all *K* extra planes, it is considered as a blocked request. Since all the blocked connections independently search for free planes among these *K* extra planes, it might be the case that multiple conflicting connections try to grab the same plane at the same time. In this case, randomly one of these connections succeeds to use the selected plane, and remaining connections continue searching for free planes. Since all connections perform their searching among *K* extra planes independently and each connection searches an extra plane at most one time, the time complexity for searching is O(*K*log*N*), and therefore, the overall time complexity of the PFR_LS algorithm remains *O*(log*N*) when *K* is constant and much smaller compared to *T*.

VI. PERFORMANCE ANALYSIS

In this section, we analyze the performance of algorithms PFR, PFR_RS and PFR_LS algorithms in terms of blocking probability and hardware cost.

A. Blocking Probability

An extensive simulation has been performed to study the blocking probabilities of P-VSOB(*N*,*T*) network and EP-VSOB(*N*,*T*+*K*) employing corresponding routing algorithms proposed above. The blocking probability of the network is defined as the probability that a free input cannot be connected to a free output due to the fact that other connections are already using a part of the path for the requested connection. Each input has equal probability. The network simulator we developed consists of four major modules as shown in Figure 5.

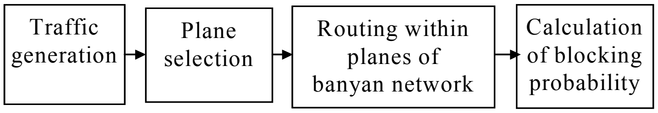


Figure 5: Block diagram of network simulator.

We consider here the permutation request as the traffic since a permutation does not have output contention, and therefore, gives real blocking probability of the switch network only. Due to the symmetric architecture of P-VSOB(*N,T*) network, every connection request has the same probability to be blocked. In our simulation, we fix the connection request of input-output pair 0-0 and investigate the blocking probability of this connection request only. The traffic generation module randomly generates a permutation request for a network based on the workload *r* (here workload *r* is defined as the occupancy probability of a port). The plane selection module attempts to assign connection requests to different planes using a routing algorithm. Connections assigned to a plane are then established by self-routing algorithm. A connection can be blocked either for link-busy or crosstalk constraint. The blocking probability is then estimated by the ratio of number of permutation requests in which the 0-0 request is blocked to the total number of permutation requests generated.

1) Blocking Probability of P-VSOB(N,T) Networks

We have examined four network configurations of *N*={256,512,1024,2048}. For each configuration, the blocking probabilities are simulated for workload, *r*={1.0,0.9,0.8} and crosstalk SE, *c*={0,1,2,3}. The corresponding results for PFR algorithm are summarized in Figures 6, 7. Table I is presented to give the reader an idea about the blocking probability in numerical figures when 3 CSEs are allowed in a signal path.

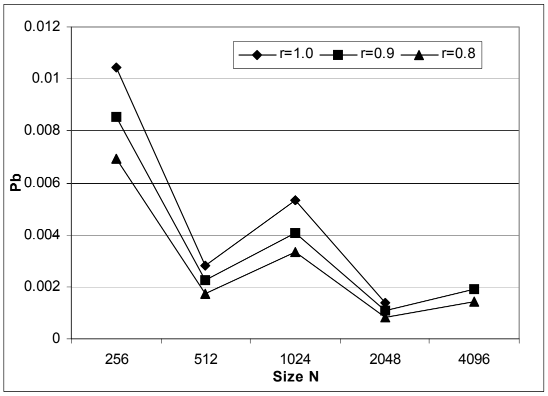


Figure 6: Blocking probabilities of pruned VSOB(N,T) networks with crosstalk, *c* = 1.

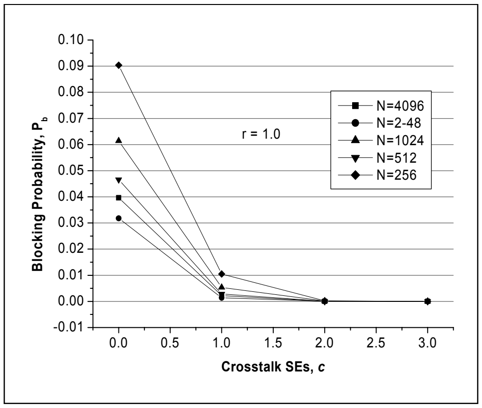


Figure 7: Blocking probabilities of P-VSOB(N,T) networks with different crosstalk constraints.

Table I: Blocking probabilities of P-VSOB(N,T) networks employing PFR algorithm with c = 3.

Size, N	Blocking probability		
	r = 1.0	r = 0.9	r = 0.8
256	0.000004	0.000002	0.000002
512	0	0	0
1024	0.000002	0	0
2048	0	0	0
4096	0	0	0

It is interesting to note from Figure 6 that though the blocking probability of a P-VSOB(N,T) network employing PFR algorithm decreases to a low value as the network size increases, this decrease in blocking probability is not monotonous. A network with an odd number of stages (i.e. log$N$ is odd) has relatively lower blocking probability than that of a network with an even number of stages (i.e. log$N$ is even). For example, the blocking probability of 512×512 network is always lower than that of 1024×1024 network. The reason is that a P-VSOB(2N,T)) network with odd number of stages has the same number of planes as that of P-VSOB(2N,T)) network. Therefore, the number of connections established in a plane of P-VSOB(N,T) is half of that for a plane in P-VSOB(2N,T). However, blocking may happen from stage (log$N$+1)/2 to stage log$N$ when log$N$ is odd while blocking may happen from stage (log$N$)/2 to stage log$N$ when log$N$ is even. For example, blocking may happen in the last 5 stages of P-VSOB(512,32) network while blocking may happen in the last 6 stages of P-VSOB(1024,32) network. Due to the higher number of stages contributing to the blocking, P-VSOB(2N,T) network has a higher blocking probability than that of P-VSOB(N,T) network when log$N$ is odd.

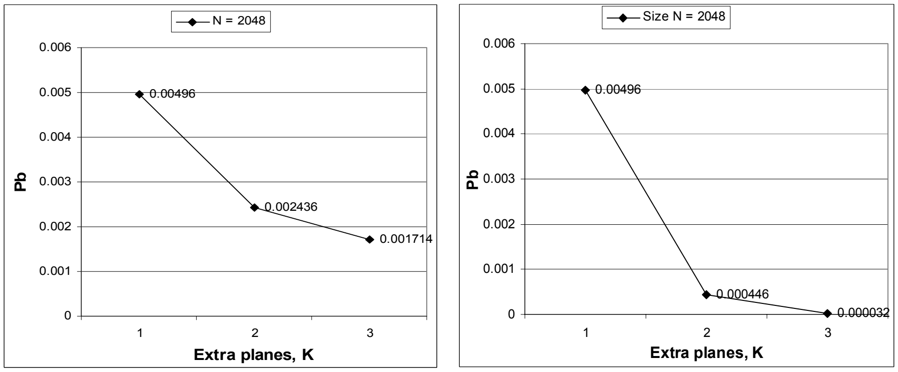
Figure 7 shows that allowing crosstalk in a signal path we can decreases the blocking probability drastically. The blocking probability is reduced to less than 0.0002% (for 1024×1024 networks at r = 1.0) when only 3 CSEs are allowed, even though the workload is 100% (as per Table I). From the practical point of view 3 CSEs along a path is quite acceptable. It is easy to see that blocking probability reduction from higher number of CSEs is not significant. Some entries in the table show zero due to limited precision of our simulation program. Blocking probabilities in those configurations are lower than 0.000001, and have been rounded up to zero.

The blocking probability of the PFR algorithm is very impressive considering the self-routing property and low hardware cost (as will be shown later) of a P-VSOB(N,T) network. To guarantee zero blocking probability in a self-routing network, usually very high hardware cost is required. For example, nonblocking Crossbar and

Spanke's networks have hardware costs O(N^2) [1][14] whereas VSOB(N,T) (or P-VSOB(N,T)) has hardware cost $O(N\sqrt{N}\log N)$.

2) Blocking Probability for EP-VSOB(N,T+K) Network

In this section, we explore the effects of using extra planes on the blocking probability in an EP-VSOB($N,T+K$) network. The simulation results are summarized in Figures 8, 9. Table II gives an idea about how low the blocking probability has been achieved.



(a) (b)

Figure 8: Blocking probabilities of EP-VSOB(2048, 64+K) networks at r = 1.0. (a) With PFR_RS algorithm and (b) With PFR_LS algorithm.

From Figure 8 it is clear that PFR_RS-based EP-VSOB($N,T+1$) networks have the same performance as that of PFR_LS-based EP-VSOB($N,T+1$) networks since there is only one extra plane and nothing to search for. However, the effect on blocking probability is drastic when K increases with PFR_LS algorithm. From Table II we see that the blocking probability is a low as 0.0008% for a 2048×2048 network .

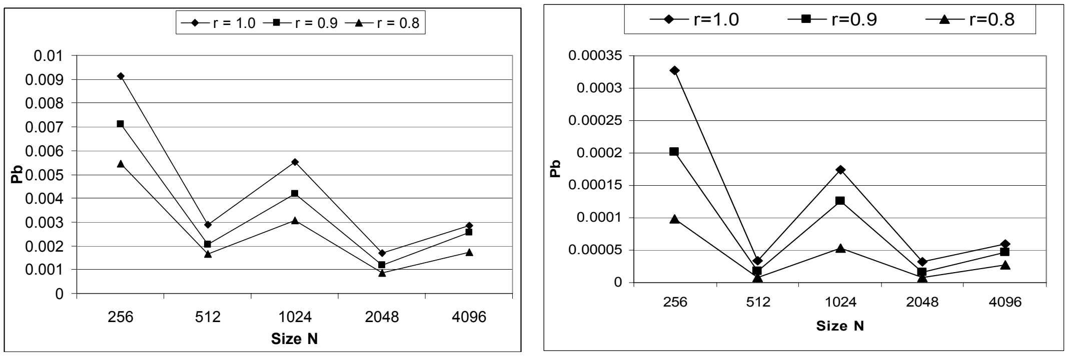
Table II: Blocking probabilities of EP-VSOB($N,T+3$) networks employing PFR_LS algorithm

Size N	Blocking probability		
	r = 1.0	r = 0.9	r = 0.8
256	0.000328	0.000202	0.000098
512	0.000048	0.000014	0.000006
1024	0.000174	0.000126	0.000054
2048	0.000032	0.000016	0.000008
4096	0.00006	0.000046	0.000027

Results in Figure 8 and Table II indicate that by adding only a small number of extra planes to the P-VSOB(N,T) networks, the EP-VSOB($N,T+K$) networks (specially the PFR_LS-based EP-VSOB($N,T+K$) networks) can reduce the blocking probability sharply. For example, when workload r=1.0, the blocking probability for the VSOB(2048,64) network is 0.03179 [10], and this blocking probability is reduced to 0.002436 in the PFR_RS based EP-VSOB(2048,64+2) network, and further to 0.000446 in the PFR_LS based EP-VSOB(2048,64+2) network. The above simulation results also indicate that a EP-VSOB($N,T+K$) network adopting PFR_LS algorithm always achieve much lower blocking probability than that of adopting PFR_RS algorithm. A PFR_LS-based EP-VSOB($N,T+K$) network is much more sensitive to the parameter K than that of PFR_RS-based EP-VSOB($N,T+K$) network, which is evident from the slopes of both curves. However, the blocking probability reduction is not significant for K>3 since the curve for PFR_LS algorithm has almost reached to zero at K = 3.

Therefore, we can safely say that an EP-VSOB(N,T+3) network with the PFR_LS algorithm is a good choice for designing optical switching networks with time complexity O(log$N$), where zero crosstalk has to be ensured.

It is interesting to note that the aforementioned [10] characteristic of odd and even stages also prevails in PFR_LS-based EP-VSOB(N,T+K) network, and the reason is the same as discussed P-VSOB(N,T) network. Two graphs in Figure 9 will help reader understand how the blocking probability changes in different configurations.



(a) (b)

Figure 9: Blocking probabilities of EP-VSOB(N,T+3) networks. (a) For PFR_RS algorithm. (b) For PFR_LS algorithm.

B. Hardware Cost

The hardware cost of a P-VSOB(N,T) network is reduced for two reasons: simpler input interface circuitry and lower switch count. Since each input in a P-VSOB(N,T) network is fixed to only one plane, no splitters/switches are required at the input side, unlike regular VSOB(N,T) networks. This also simplifies the control circuitry. Again, since only one input from each input group is used, other input switches and switches in their successive stages are also not used.

From Figure 3 it is easy to see that an input group forms a $T \times T$ network. Again, this $T \times T$ network consists of two $T/2 \times T/2$ subnetworks shuffled with T switches. One of these $T/2 \times T/2$ subnetworks does not carry any signals, therefore, all switches belonged to this $T/2 \times T/2$ subnetwork are redundant. The other $T/2 \times T/2$ subnetwork can also be divided into two $T/4 \times T/4$ subnetworks, in which one is redundant. Proceeding in this way the total number of redundant switches in one input group is given by following equation,

$$R_g = \frac{T}{2} . \frac{1}{2}\left(\log\left(\frac{T}{2}\right)\right) + \frac{T}{4} . \frac{1}{2}\left(\log\left(\frac{T}{4}\right)\right) + ... + 4 + 1.$$

$$= \sum_{i=1}^{\log T - 1} i 2^{i-1}$$

The total number of redundant switches in the P-VSOB(N,T) is

$$R_{VSOB} = N \sum_{i=1}^{\log_2 T - 1} i 2^{i-1} \tag{3}$$

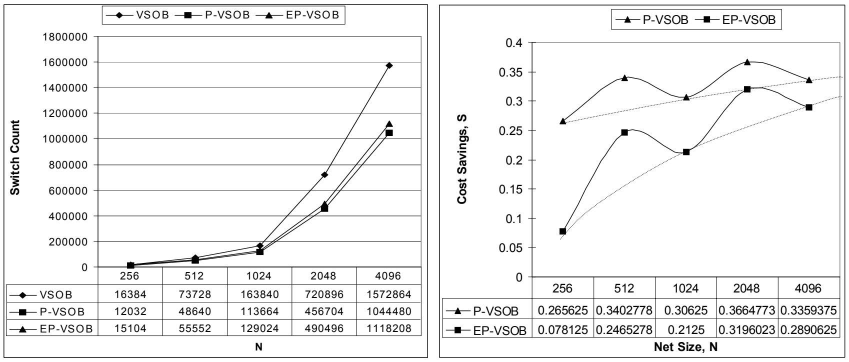
Therefore, the switch count for P-VSOB(N,T) networks is,

$$T_{\text{Pruned VSOB}} = T\left(\frac{N}{2}\log N\right) - N\sum_{i=1}^{\log T-1} i2^{i-1} \tag{4}$$

Note that for a regular VSOB(*N*,*T*) network, the switch count is given by:

$$T_{N(\text{Regular VSOB})} = T\left(\frac{N}{2}\log N\right)$$

Figure 10 compares the switch counts of regular VSOB*(N,T),* P-VSOB*(N,T)* and EP-VSOB*(N,T+K)* networks. It is clear that the P-VSOB network saves the number of SEs significantly, and this savings of SEs increases with the increase of the network size. The difference between the cost savings of P-VSOB(N,T) and EP-VSOB(N,T) gradually decreases as the network size increases. For example, when N=256, $S_{\text{P-VSOB}}$ is 26.5% and $S_{\text{EP-VSOB}}$ is 7.8%, whereas for N = 4096, $S_{\text{P-VSOB}}$ is 33.5% and $S_{\text{EP-VSOB}}$ is 28.9%.



	256	512	1024	2048	4096
VSOB	16384	73728	163840	720896	1572864
P-VSOB	12032	48640	113664	456704	1044480
EP-VSOB	15104	55552	129024	490496	1118208

	256	512	1024	2048	4096
P-VSOB	0.265625	0.3402778	0.30625	0.3664773	0.3359375
EP-VSOB	0.078125	0.2465278	0.2125	0.3196023	0.2890625

Figure 10: Comparison of VSOB, Pruned VSOB and EVSOB networks on Switch Count. (a) Switch count versus Network size. (b) Switch count savings in ratio with regular VSOB networks.

In case of EP-VSOB(*N*, *T*+*K*) networks, as we use *K* extra planes with the original *T* planes, and each extra plane is an *N*×*N* regular banyan network, the switch count of EP-VSOB(*N*,*T*+*K*) is

$$T_N = (K+T)\left(\frac{N}{2}\log N\right) - N\sum_{i=1}^{\log T-1} i2^{i-1}\ . \tag{5}$$

Since *K*<<*T*, the hardware cost of a EP-VSOB(*N*, *T*+*K*) network is still much less compared to a regular VSOB(*N,T*) network, as seen in Figure 10. The cost savings are calculated as following:

$$S_{P-VSOB} = \frac{N\sum_{i=1}^{\log T-1} i2^{i-1}}{\frac{NT}{2}(\log N)} \tag{6}$$

$$S_{EP-VSOB} = \frac{N\sum_{i=1}^{\log T-1} i2^{i-1} - \frac{KN}{2}(\log N)}{\frac{(K+T)N}{2}(\log N)} \tag{7}$$

(6)

Fig 10 (b) shows that the cost savings for networks having odd number of stages are always higher than that of networks having even number of stages. It also reveals that the difference between cost savings of P-VSOB(N,T) and EP-VSOB($K+T, N$) is reduced as the network size increases.

VI. CONCLUSION

In this paper, we have studied the problem of fast connection establishment in VSOB networks. We first proposed pruning of VSOB networks which still works with the PFR algorithm but reduces hardware cost by almost 30%. We have analyzed the blocking probability of P-VSOB networks with different crosstalk constraint and found that only 3 crosstalk SEs along a signal path can reduce the blocking probability to a very low value (<0.0004%). Noting that the blocking probability of a P-VSOB network adopting the PFR algorithm may be considered too high (with zero crosstalk constraint) in some high performance applications, we further proposed the EP-VSOB network that has potential to reduce the blocking probability dramatically even with zero crosstalk constraint. To guarantee the fast connection setup in the EP-VSOSB network, we also proposed two PFR-based routing algorithms for it, namely PFR with random selection (PFR_RS) algorithm and PFR with linear search (PFR_LS) algorithm. Extensive simulation results indicate that by adding only a small number of extra planes (say three) to a P-VSOB network, the EP-VSOB network combined with its two routing algorithms (in particular, the PFR_LS algorithm) has the capability to achieve very low blocking probability while keeping the optimum routing complexity of O(log$N$). Hardware costs of EP-VSOB networks are comparable to P-VSOB networks and much lower than that of VSOB networks. We believe both the approaches to reducing the blocking probability proposed in this paper will make the adoption of banyan-based optical switches very attractive in most practical applications.

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