Pure appl. geophys. 158 (2001) 919–930
0033–4553/01/060919–12 $ 1.50 + 0.20/0

Pure and Applied Geophysics

Intermittent Phase Transitions in a Slider-block Model as a Mechanism for Earthquakes

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Abstract — The aperiodic behavior of a two-dimensional stick-slip slider-block model of earthquakes is the subject of this study. The phenomenon of temporal phase transitions between creep and stick-slip motions is thoroughly analyzed and attributed to type-I intermittency. Asymmetry in the elastic forces is shown to play a key role in the emergence of complex behavior in the model. The unpredictability of chaotic bursts in the intermittent regime indicates potential difficulties for earthquake forecasting.

Key words: Stick-slip, friction, earthquakes, oscillators.

1. Introduction

The conjecture that complicated spatio-temporal patterns of seismic activity may originate from the chaotic dynamics of tectonic plates at their interface has recently attracted considerable attention. Various models comprising blocks, springs and nonlinear friction have been proposed to clarify the basic mechanisms underlying the apparent irregularity of seismic events (BURRIDGE and KNOPOFF, 1967; DIETERICH, 1978, 1979; ITO, 1980; RUINA, 1983; RICE and TSE, 1986; NUSSBAUM and RUINA, 1987; CARLSON and LANGER, 1989; HUANG and TURCOTTE, 1990a, b; GU and WONG, 1991; HE *et al.*, 1988). An important motivation for many of these studies emanates from the hypothesis that if the dynamics of earthquakes is indeed a deterministically chaotic process then it appears possible to develop an efficient short- or intermediate-time prediction algorithm for large earthquakes (KEILIS-BOROK, 1997).

A simple mechanical system that can serve as a prototype in modeling seismicity is shown in Figure 1a. It consists of the block of mass m attached by springs of stiffness $k_{x,y}$ to the motionless walls. The bottom of the block is in contact with a surface moving slowly (at a speed $\vec{\alpha}$) in an arbitrary direction. The main source of complexity in this class of models is a nonlinear dry friction law which in the simplest case is just velocity weakening. This means that, after the static friction force between the bottom of the slider and the surface exceeds a

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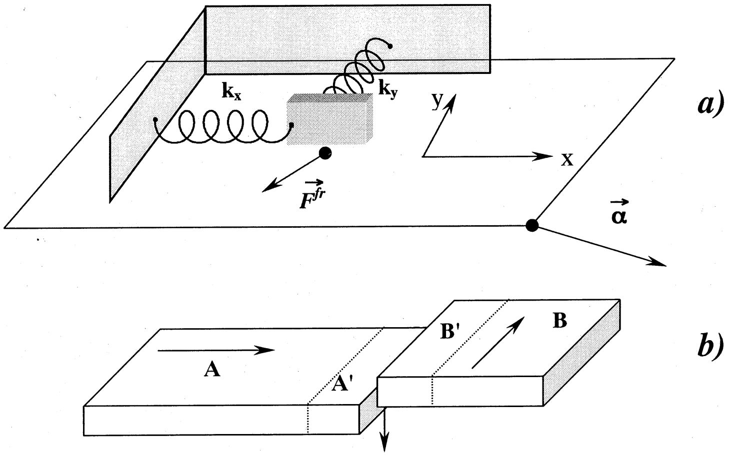

Figure 1
(a) Mechanical model of earthquakes. (b) Schematic of three-dimensional configuration of tectonic plates, e.g., in the vicinity of a subduction zone. Tectonic plate A moves to the left, submerging under plate B also moving along the fault. Part B′ (counterpart of the block in our model) of plate B experiences complicated deformations which are substantially different vertically (low elastic rigidity) and horizontally (high rigidity).

certain threshold, the block begins to slide. In the sliding phase, the higher its velocity the lower the (now dynamic) friction force. It is now well-established (SHAW, 1995; SCHOLZ, 1990) that this kind of friction-velocity relation is sufficient to produce periodic stick-slip oscillatory behavior. It should be noted that similar behavior of the slider can be obtained with other types of friction laws such as slip weakening (SHAW, 1995) or rate- and state-dependent friction law (DIETERICH, 1978, 1979; RUINA, 1983). By the introduction of additional degrees of freedom as state variables, the latter also makes it possible to reproduce various complicated oscillatory regimes (GU and WONG, 1991), which have been observed in experimental situations with different block and substrate materials. However, this model is intrinsically high-dimensional, since it is *a priori* unclear how many state variables should be used to describe the friction force. Therefore, the dynamics of the model proposed by Dieterich and Ruina is strongly dependent on the controls: this makes the analytical treatment complicated or impossible, and in many cases does not allow an unambiguous interpretation of experimental results in terms of simulation parameters.

In the present article, we propose a generalized stick-slip model of earthquake dynamics. Contrary to previous studies, we concentrate not on the complexity in the behavior resulting from rate- and/or state-dependent friction, but consider a dynamic mechanism for chaotic motion, based on the asymmetry of elastic forces and the two-dimensional character of the problem. This approach reveals a new mechanism of switching between creep and large amplitude stick-slip motion, which might be important for modeling earthquakes. We stress that the intermittent mechanism discussed below is almost independent from the details of the friction law. The only factor used in the analysis is the presence of instability (velocity weakening in the study) at the initial stage of motion. Note also, that most spring-block models studied to date were in fact one-dimensional, although the problem at hand has at least two physical dimensions, for it corresponds to the processes at the surface of contact between two tectonic plates. Even in the two-dimensional numerical simulation of the multiblock fault model performed by CARLSON (1991), each block was allowed to move only in a single direction of the driving force.

As an illustrative example of one possible configuration in which asymmetry in the elastic forces may play a crucial role in the seismic dynamics, we show in Figure 1b a hypothetical situation within a subduction zone, where a plate, A, submerges under a plate, B, while the latter also experiences a lateral displacement along the fault. Taking into account the rigidity of the lithosphere, which may be several orders of magnitude higher in the horizontal direction than that in the vertical one, it becomes clear that part B' of plate B experiences approximately the same, highly asymmetric, distribution of elastic forces as the block shown in Figure 1a.

It has been recently shown by VASCONCELOS (1996), in the case of seismic activity, when the magnitude of the velocity $|\vec{\alpha}|$ tends to zero, the one-dimensional model can demonstrate two basic types of motion: creep and stick-slip, the former being also of stick-slip type, however of infinitesimally small amplitude. The most interesting feature found by Vasconcelos is the phenomenon similar to phase transitions of the first order between these two types of behavior, when the parameter δ (defined below) that controls the intrinsic instability of the friction law is varied. However, since the model of VASCONCELOS (1996) is one-dimensional, it lacks a mechanism for temporal transition between the phases, therefore, the model is unable to demonstrate complex behavior at fixed values of the control parameters. We show that, if a second degree of freedom is introduced to the geometry of the model, the heterogeneity of the restoring elastic forces can result in intermittent switching between creep and stick-slip oscillatory regimes. The presence of a second degree of freedom in our model makes the generalized instability parameter, which controls the phase transition, dependent upon the coordinates of the sliding block. As the coordinates change with time, this property provides a natural mechanism for temporal phase transitions between creep and stick-slip phases of motion.

The importance of anisotropy in the parameters of stick-slip oscillators for producing complicated temporal dynamics was first recognized by HUANG and TURCOTTE (1990a,b), who analyzed a two-block configuration in a seismological context. The presence of asymmetry was claimed to be a necessary condition for chaotic regimes to appear in this system. Although it was later demonstrated by RYABOV and ITO (1995), DE SOUSA VIEIRA (1999), that this property is not necessary for complexity to manifest itself, it is undoubtedly one of the most important factors to be considered for the analysis of interaction between tectonic plates.

2. *Mathematical Formulation of the Problem*

We start with the equations of motion for the two-dimensional slider shown in Figure 1

$$
\begin{aligned}
m\ddot{x} &= -k_x x + F_x^{fr} \equiv -k_x x + |\vec{F}^{fr}|\cos(\varphi^{fr}) \\
m\ddot{y} &= -k_y y + F_y^{fr} \equiv -k_y y + |\vec{F}^{fr}|\sin(\varphi^{fr})
\end{aligned}
\tag{1}
$$

where x and y are coordinates of the block of mass m (in the following, by rescaling the time coordinate, we can always put $m = 1$), dot denotes time differentiation, $k_{x,y}$ stand for stiffness coefficients of the springs along x and y axes, $F_{x,y}^{fr}$ are the Cartesian components of the friction force of magnitude $|\vec{F}^{fr}|$ and the direction defined by the angle φ^{fr}. We assign the amplitude of the friction force to be

$$
|\vec{F}^{fr}| = F_0 \Phi(|\vec{v}_r|) \tag{2}
$$

where $|\vec{v}_r| = \sqrt{(\dot{x} - \alpha_x)^2 + (\dot{y} - \alpha_y)^2}$ is the relative velocity of the slider with respect to the substrate, $\alpha_{x,y}$ are the components of the substrate velocity, $\Phi(|\vec{v}_r|)$ is an arbitrary continuous monotonically decreasing function of the relative velocity satisfying the condition

$$
\Phi(0) = 1; \quad \Phi'(0) = -\gamma/F_0 \tag{3}
$$

where prime denotes the differentiation with respect to velocity. In fact, equations (2, 3) define two important physical parameters of the problem: the threshold value of the dry friction, F_0, and the instability rate γ imposed by the friction force at the initial phase of slip motion. A substantial difference of the present model compared to its one-dimensional counterpart consists in the necessity to take into account the direction of the friction force. Generally, in the slip phase of motion, friction is thought to be directed in the opposite direction to the velocity. At the same time, when the block is stuck, friction balances the restoring elastic force, hence, pointing to the reverse direction of the elastic force. Therefore, when the block starts to move, the orientation of the friction force should be somehow switched to the direction opposite the velocity. If the model is one-dimensional, the elastic force and velocity are collinear and the direction of the friction does not change as the slip starts. However, in a two-dimensional state, the immediate switching in the directivity of friction can cause a jerk in a certain direction at the very beginning of the slip phase of motion. The importance of temporal changes in the direction of friction force has been recently recognized in the analysis of a 3-D rupture process by MADARIAGA *et al.* (1998). They also stressed the necessity to introduce slip-weakening at the beginning of the slip event. To preserve the continuity of the block motion, we assume that at the initial stage of the slip phase, the friction force is directed opposite of the elastic force. As the velocity of the block increases, the direction of the friction force is changed to the direction opposite the velocity, otherwise the block may never reach the stick stage again. We therefore state that when the block is accelerated, the angle between the vectors of friction force φ^{fr} and stress φ^{el} is defined by

$$\begin{aligned} \varphi^{fr} &= -\varphi^{el} + \Delta\varphi[1 - \exp(-\Omega|\vec{v}_r|)] \\ \Delta\varphi &= \varphi^{el} - \varphi^{v_r} \end{aligned} \tag{4}$$

where φ^{el} and φ^{v_r} are the directions of the elastic force and relative velocity respectively, and Ω is the parameter controlling the process of switching the direction. Basically, the parameter Ω describes the memory effect in the friction law. (It is well known from many rock friction experiments that the slider at the initial stage of motion "keeps the memory" of the previous stick condition.) In the deceleration phase of the block motion, the friction force direction is fixed to the value attained at maximum velocity, otherwise the block may never reach the stick state. So defined, the friction law is, in fact, both rate- and state-dependent, and such a formulation is consistent with that of Dieterich and Ruina; the parameter $\Delta\varphi$ here playing the role of a state variable θ. Note however, that in our formulation, the state variable does not produce an additional degree of freedom to the problem, as it is defined not by a separate evolution equation, but by an explicit relation (4). Our approach is also close in spirit to that used in COCHARD and MADARIAGA (1996); MADARIAGA *et al.* (1998), who applied different (state- or rate-dependent) friction laws at different stages of the sliding process. Note also, that some of previously performed one-block simulations (see, e.g., GU and WONG, 1991), although called one-degree-of-freedom, were, in fact, several degrees of freedom dynamical systems, since state variables of the friction law introduced additional degrees of freedom to the problem.

3. Details of the Dynamics

From a general viewpoint, the system (1) consists of two couples oscillators corresponding to the motions of the block along x and y axes. Qualitatively, its dynamics are little different from that of two stick-slip oscillators (HUANG and TURCOTTE, 1990a, b; RYABOV and ITO, 1995; GALVANETTO *et al.*, 1995), although certain differences exist. As we are mainly interested in modeling seismicity, we shall not consider various oscillatory regimes which occur at large values of driving velocity (ELMER, 1997), and concentrate on the limit $|\vec{\alpha}| \to 0$. Our analysis reveals that when $|\vec{\alpha}| \ll 1$ the system can demonstrate two qualitatively different types of behavior (see also VASCONCELOS, 1996), to be further referred to as *creep* and *strong* motions. The distinction between the two oscillatory regimes consists in the amplitude of stick-slip motion experienced by the block, the former being of the order of $|\vec{\alpha}|$, i.e., of very small amplitude, while the latter being independent of $|\vec{\alpha}|$ and of comparatively large amplitude. When the control parameters are varied, motion of the block can change between periodic, quasiperiodic, and chaotic, depending on the initial conditions and parameter values. It should be noted that either creep or strong motion can be of any type, e.g., a chaotic creep or periodic strong, or any other combination. In our view, the most interesting chaotic regime observed in this system includes both creep and strong motions: first, because it occupies a considerable area in the control parameter space, and, second, it provides an insight to the process of switching between creep and strong motion, characteristic of earthquake activity on a well-developed fault. A feature of this switching process is that it happens due to the intrinsic dynamics of the system, without any variation of the controls or external conditions.

Let us now consider how a chaotic motion appears in this system. The stick-state boundary (SSB), i.e., the line on the X-Y plane where the elastic force equals the maximum static friction value, plays an important role in the dynamics of the system. It is defined by the equation

$$(k_x x)^2 + (k_y y)^2 = F_0^2 \ . \tag{5}$$

After the block reaches the SSB, it begins to slip. However, a characteristic amplitude of the slip motion depends on the position of the starting point on the SSB. Indeed, if to introduce a local rotated coordinate frame (U, V) with abscissa directed along the elastic force, and the origin located at an arbitrary point on the SSB, (x_{SSB}; y_{SSB}), then, for the initial (linear) stage of block motion the linearized version of equation (1) is

$$\begin{aligned} \ddot{u} &= -\left(\frac{k_x^3 x_{\mathrm{SSB}}^2}{F_0^2} + \frac{k_y^3 y_{\mathrm{SSB}}^2}{F_0^2}\right)u \\ &\quad + \frac{k_x k_y x_{\mathrm{SSB}} y_{\mathrm{SSB}}}{F_0^2}(k_y - k_x)v + \gamma|\dot{u} + \alpha_u| \\ \ddot{v} &= 0 \end{aligned} \tag{6}$$

where α_u is the projection of the velocity vector $\vec{\alpha}$ to the U axis. The solution of the second equation of (6) is simply $v = -\alpha_v t$. After substituting it into the first equation, and introducing the variable z as $z = u + u_0 + \mu t$, parameters

$$u_0 = \frac{\gamma}{A^2}(v - \alpha_u A); \quad \mu = \frac{v}{A};$$
$$A = \frac{k_x^3 x_{\mathrm{SSB}}^2 + k_y^3 y_{\mathrm{SSB}}^2}{F_0^2}; \quad v = \alpha_v \frac{k_x k_y x_{\mathrm{SSB}} y_{\mathrm{SSB}}}{F_0^2}(k_x - k_y)$$

then, finally, performing algebraic transformations, we obtain instead of (6) the equation of a linear oscillator with respect to the new variable z

$$\ddot{z} - \gamma\dot{z} + Az = 0 \ .$$

This equation has been studied in detail by VASCONCELOS (1996). The most important conclusion derived in this work is that the character of the solution in Eq. (6) is defined by the parameter $\delta \equiv \gamma^2/(4A)$, and in the limit $|\vec{\alpha}| \to 0$ the amplitude of stick-slip oscillations vanishes if $\delta < 1$, whereas it remains finite for

$\delta > 1$. As the parameter δ passes through its critical value of unity, the system undergoes a phase transition of the first order, switching from the creep motion to the large amplitude stick-slip.

In the two-dimensional model, the dynamics depends on the asymmetry of the elastic force. If it is symmetric, i.e., $k_x = k_y$, then $A = k_x = k_y$, the parameter δ becomes independent of the coordinates of the block, and the model is intrinsically one-dimensional. If, however, asymmetry is present, then the amplitude of the block motion in the slip phase depends on its start position on the SSB ellipse. If, in addition, the direction of the velocity $\vec{\alpha}$ is chosen so that the block creeps along the SSB, then a chaotic attractor can appear in the phase space of the system.

A fragment of a typical chaotic trajectory of the block on the X-Y plane corresponding to a characteristic cycle between two large slip events is shown in Figure 2a. After a previous large slip event the block sticks and is pulled by the lower surface to the SSB, reached at the point A. If the value of the parameter δ is less than one at this point, a small slip event occurs, the block slips, then sticks at the point A′ (Fig. 2b). Then it is pulled back to the SSB, and slips again from the point A″. This recurrent motion (creep) continues until the critical point (B in Fig. 2a) is reached, where $\delta = 1$. Here the phase transition occurs, and the next slip event has a large amplitude. In the time domain, the motion looks reminiscent of the phenomenon of intermittency, since the block is most often either stuck or in the creep phase of motion

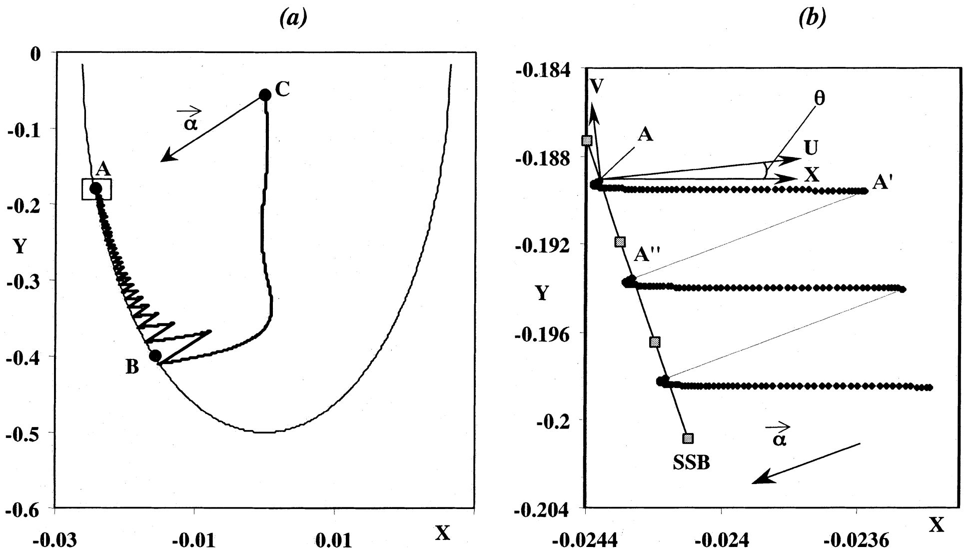

Figure 2
(a) Chaotic trajectory of the block on the XY plane at $m = 1$, $k_x = 38$, $k_y = 2$, $\gamma = 7$, $F_0 = 1$, $\Omega = 50$, $\alpha_x = -2.67 \times 10^{-4}$, $\alpha_y = -9.63 \times 10^{-4}$. (b) Enlarged area in the vicinity of point A (creep motion). Unmarked line corresponds to the stick state of the block, circles mark the slip phase of motion, squares denote stick state boundary line.

interrupted by large slip events. We attribute this regime to type-I intermittency, using the classification proposed by POMEAU and MANNEVILLE (1980). In Figure 3, a probability distribution for time intervals between two consecutive large events is plotted. A characteristic feature of this distribution is a clear cutoff at large values of interevent times, typical of type-I intermittency (BERGE *et al.*, 1984).

It should be noted, however, that, contrary to the conventional type-I intermittency found in many physical systems, the intermittent behavior in this model is an intrinsic property of the system, in the sense that it occupies a large area in the control parameter space, not only a small strip adjacent to a saddle-node bifurcation. Moreover, as we show below, it is not related to any saddle-node bifurcation.

To gain further insight into the intermittent behavior of the model, we consider the one-dimensional mapping that describes the dynamics of the system (see also GALVANETTO and KNUDSEN, 1997). We use for this purpose the transformation of the accessible part of the SSB to itself during one stick-slip cycle. Figure 2b shows the relationship between the position of points like A and A″ on the XY plane for any point that belongs to the SSB and can be reached by the block. It is convenient to express this function in terms of the variable $\theta = \arctan[k_y y_{\mathrm{SSB}}/(k_x x_{\mathrm{SSB}})]$, defining the orientation of the elastic force at the SSB. The map is obtained by numerically integrating Eq. (1) for one slip and stick cycle and all the initial conditions located on the SSB. An example of the resulting map is given in Figure 4. Note that it possesses two necessary ingredients for intermittent motion: a narrow channel between the graph and diagonal, where the phase trajectory evolves during a laminar phase of intermittent behavior, and the part responsible for reinjection of the phase trajectory to the channel. For the present model, the laminar behavior corresponds to the creep motion of the block, whereas irregular bursts of high amplitude are, in fact, large slip events (phase transitions).

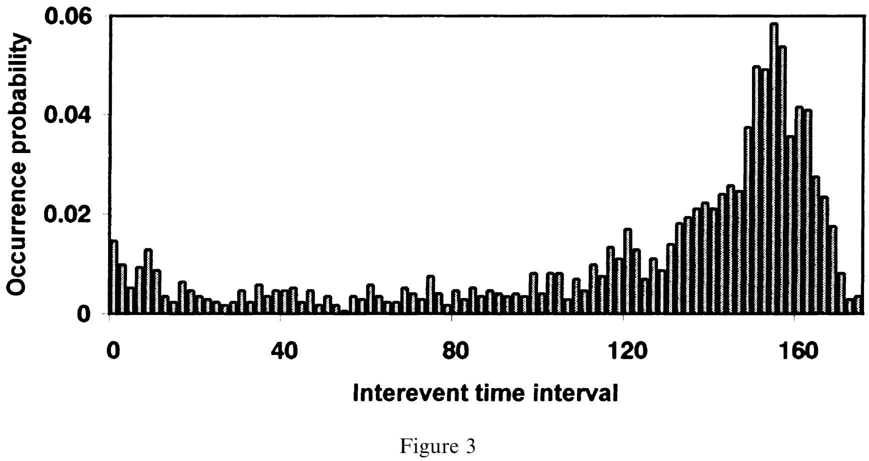

Figure 3
Histogram of time intervals between successive earthquakes (distribution of laminar phases in the intermittent regime). Parameters are the same as in Figure 2, except $\alpha_x = -2.67 \times 10^{-5}$, $\alpha_y = -9.63 \times 10^{-5}$.

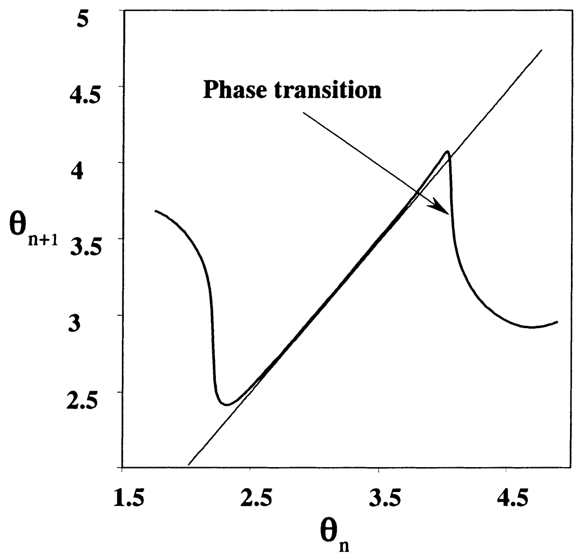

Figure 4
Intermittent one-dimensional map for the system (1) for the parameter values corresponding to Figure 2. If $|\vec{\alpha}| \to 0$ the width of the channel between the map and diagonal vanishes and the slope at the phase transition point becomes vertical.

The width of the channel is defined by the amplitude of slip events in the creep phase, which, in its turn, is proportional to the velocity of slow motion $|\vec{\alpha}|$. The value of the parameter $|\vec{\alpha}|$ has been estimated by CARLSON and LANGER (1989); VASCONCELOS (1996), to be of the order of 10^{-9} in the case corresponding to earthquakes. Therefore, the channel should always be narrow, not because the system is close to saddle-node bifurcation, but due to the extermely small value of $|\vec{\alpha}|$. This means that, as far as $|\vec{\alpha}| \ll 1$, intermittency is a typical phenomenon for this particular model, if other parameters are properly tuned to ensure the reinjection of phase trajectory into the channel.

4. *Discussion and Conclusion*

To conclude, we would like to outline certain important consequences for earthquake analysis, ensuing from the model described above. It is first interesting to note that during much of the time between consecutive large earthquakes, shear stress may be close to its critical value. For example, in Figure 5 we plot the time evolution of stress and energy regarding the intermittent character of the dynamics. Note that shear stress (elastic force) is close to its maximum and is kept almost constant during the creep phase of motion, while the amount of accumulated energy continues to grow as the block moves along the SSB to the point of phase transition

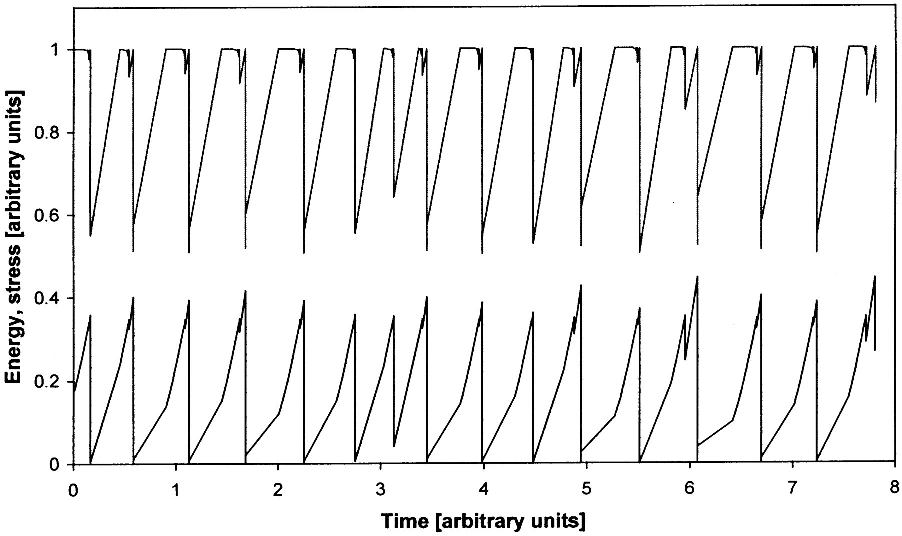

Figure 5
Time evolution of shear stress (upper curve) and elastic energy (lower curve) accumulated in springs for the intermittent regime of switching between creep and large amplitude motions.

(earthquake). Qualitatively this means in terms of shear stress that the earthquake cycle, defined as the time between two consecutive large earthquakes, consists of three parts. In the first part, stress and energy grow linearly with time, which corresponds to the classical elastic rebound theory. In the second stage, which may be approximately of the same duration as the first, stress is nearly constant, while energy continues linear (or almost linear) growth until the critical point. The third phase is a fast stress and energy drop corresponding to an earthquake. Therefore, quite contrary to the common view, the observation that shear stress has reached a threshold value may not be evidence of an approaching earthquake.

Another feature of the model is the presence of a maximum in the distribution of interevent time intervals (Fig. 3). This property is consistent with the theory of characteristic earthquakes, which states that large seismic events are approximately periodic. As there is no exact periodicity in any sequence of natural earthquakes, the intermittent chaotic regime in the proposed model may be a good simulation of seismic activity at a separate fault or fault segment.

Finally, if type-I intermittency is indeed the mechanism for real earthquakes, then, contrary to expectations mentioned in the beginning of this article, it appears impossible to produce an effective prediction of the next large slip event by applying the group of methods based on the concept of embedding and correlation dimension analysis (WEIGEND and GERSHENFELD, 1993). For example, in Figure 6 we plot a return map for time intervals between to successive large events. The absence of apparent clustering of points on this plot indicates poor predictability in the time domain. The

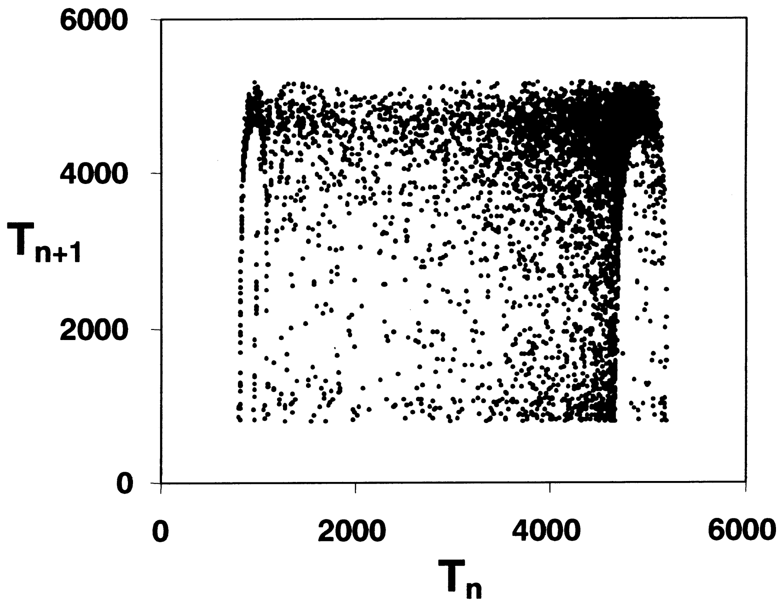

Figure 6
Time interval between successive earthquakes vs. the same characteristic calculated for two previous events. Values of control parameters correspond to Figure 3.

main reason for this is the sharp drop of the local map function in the vicinity of the phase transition (reinjection) point, that renders the expansion rate of the phase flow during large events extremely high. This means that the prediction produced for this model by any method from the dynamical systems theory discussed, e.g., in WEIGEND and GERSHENFELD (1993), would not be better than that for a linear stochastic process.

Acknowledgements

This work has been partially supported by INTAS and JSPS. We thank Professor A. Correig, who reviewed the manuscript, and Dr. T. Ouchi for his interest in this work and important discussions.

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(Received December 20, 1999, accepted May 29, 2000)